From finance to fisheries – Using market models to evaluate returns versus risk for ESA-listed Pacific salmon

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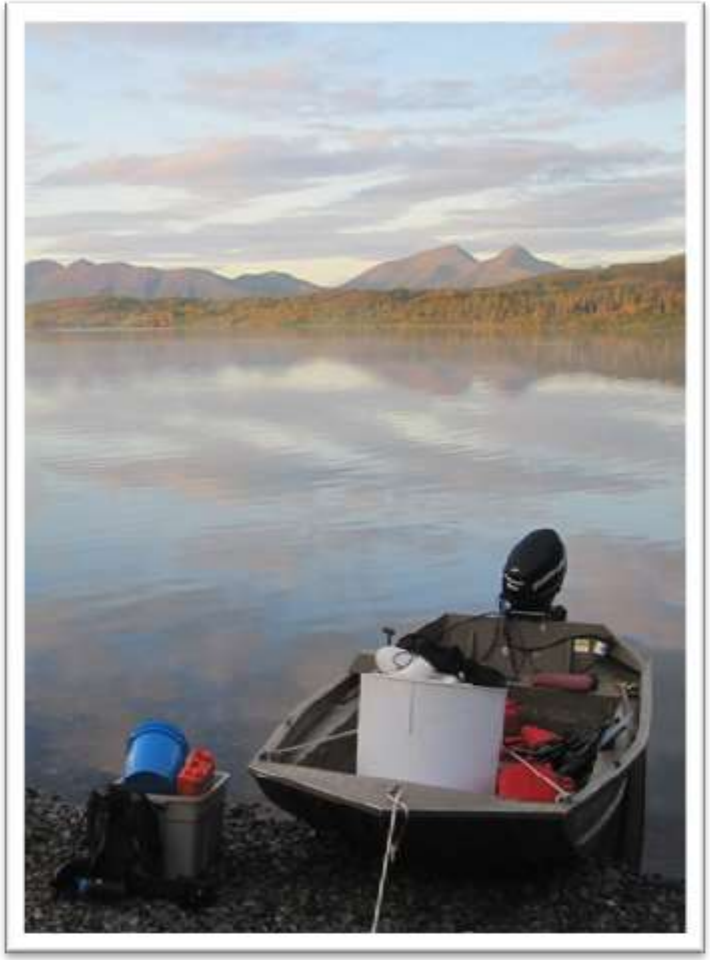
Outline



- Notions of risk in finance
- Capital Asset Pricing Model (CAPM)
- Conservation analogues
- Dynamic Linear Models (DLMs)
- Salmon case study
- Next steps

Notions of risk in finance

- Financial markets are rife with various forms of risk
- For simplicity, let's consider 2 broad categories:

1) Systematic (market) risk is vulnerability to large-scale events or outcomes that affect entire markets (eg, natural disasters, govt policy, terrorism)

2) Unsystematic (asset) risk is specific to particular securities or industries (eg, droughts affect commodities like corn but not oil; bad batteries affect Boeing but not Microsoft)

Diversification

- By holding a diverse collection of assets (a portfolio), one can reduce unsystematic, but not systematic, risk
- Investing is inherently risky, but (rationale) investors are risk averse
- That is, if presented with 2 portfolios offering equal returns, they should choose the less risky one
- Thus, investors expect to be compensated with higher returns for accepting more risk & vice-versa

Estimating risks

- Portfolios can only reduce unsystematic risks, so one should understand the systematic risk of an asset before it is added to a portfolio
- Sharpe (1963) outlined a model whereby returns of various assets are related through a combination of a common underlying influence & random factors
- The total variance in returns of a particular asset equals the variance in larger market returns plus residual variance uncorrelated with the asset
- Total risk = Systematic risk + Unsystematic risk

The market model

- Many others (e.g., Treynor, Lintner, Beja) were also working on these ideas, which ultimately led to the "market model"

Returns of asset at time *t*

Returns of market at time *t*

$$r_{a,t} = a_a + b_a r_{m,t} + v_{a,t}$$

Manager's skill

Correlated volatility (sensitivity to systematic risk)

Interpreting alpha

Value of α	Interpretation
$\alpha < 0$	Asset earns too little for its risk

*

*Expected value if market is "efficient" (*sensu* Fama 1970)

Interpreting beta

Value of β	Interpretation	Example
$\beta = 0$	Movement of asset is independent of market	Fixed-yield bond

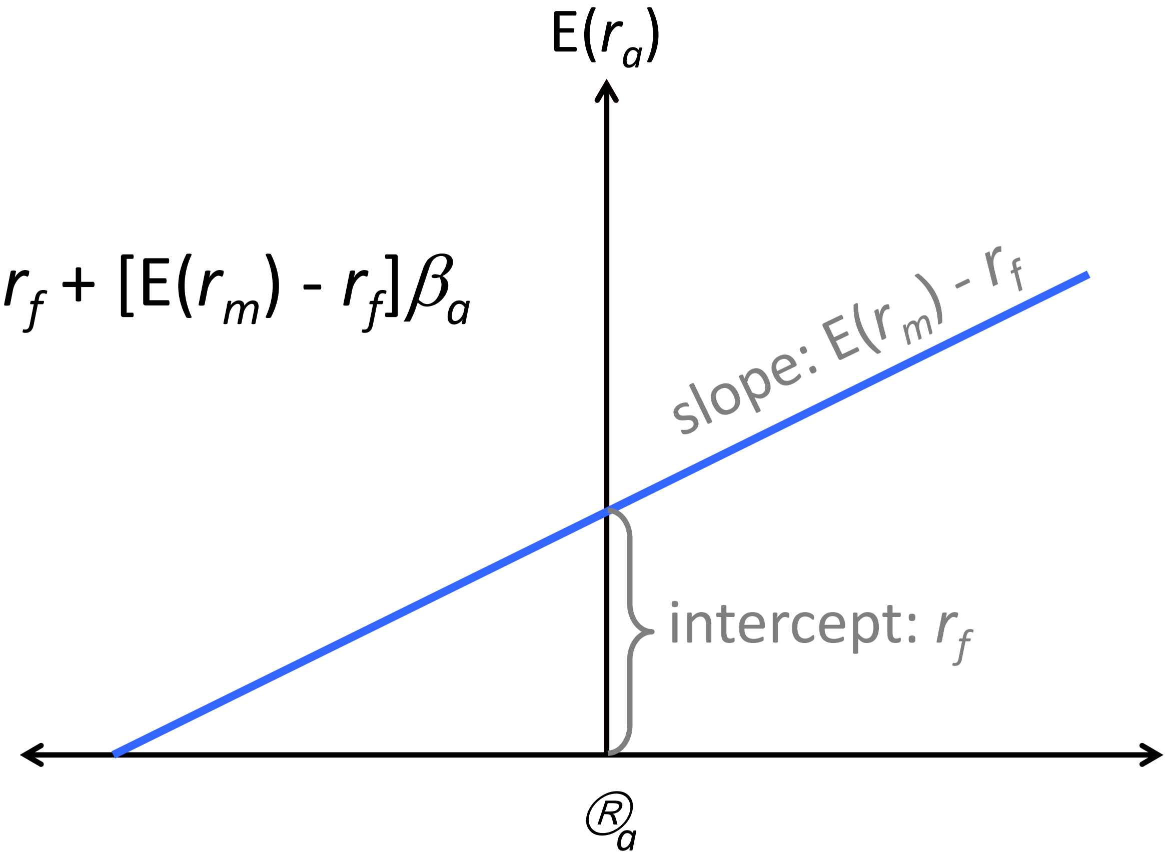
Capital Asset Pricing Model (CAPM)

- CAPM followed directly from the market model
- CAPM determines an expected rate of return necessary for an asset to be included in a portfolio based on:
  1) the asset's responsiveness to systematic risk (β);
  2) the expected return of the market; and
  3) the expected return of a risk-free asset (eg, US govt T-bills)
- CAPM is usually expressed via the security market line:

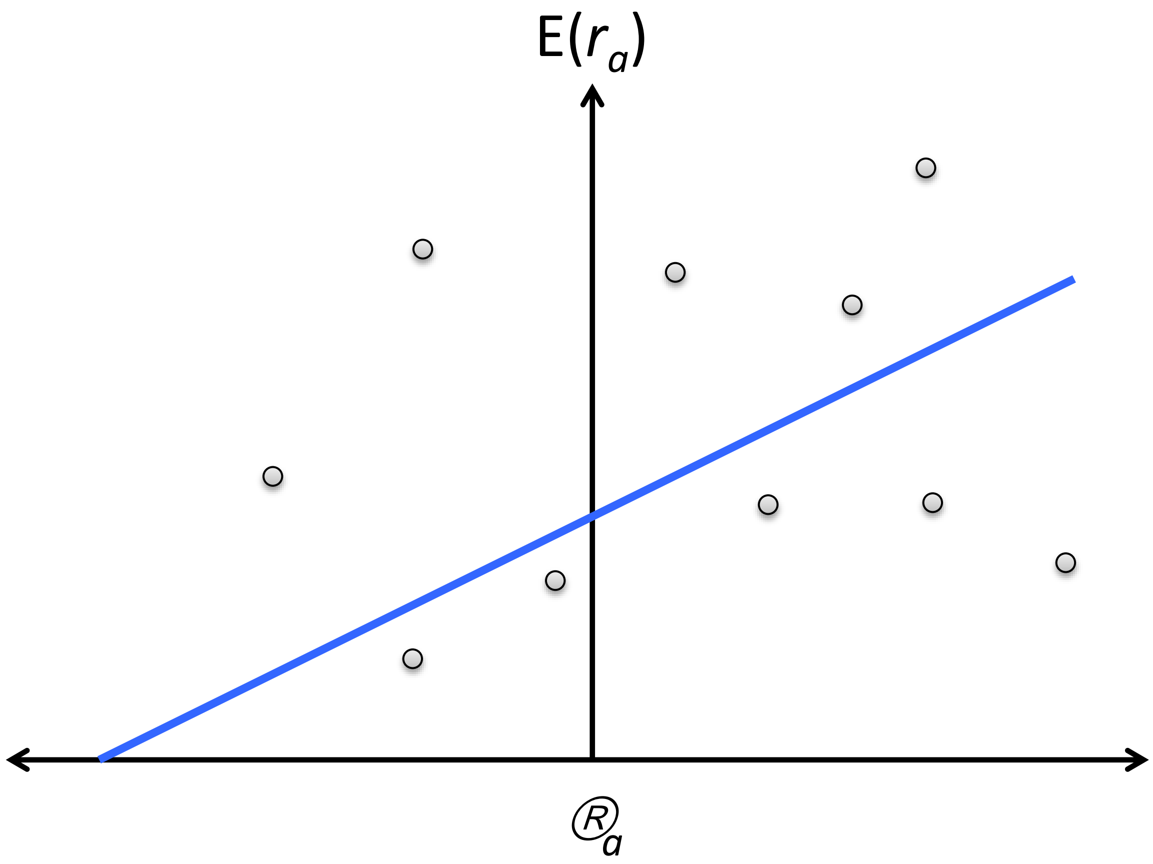
$$\mathrm{E}\left(r_a\right) = r_f + \left[\mathrm{E}\left(r_m\right) - r_f\right]\beta_a$$

Security market line (SML)

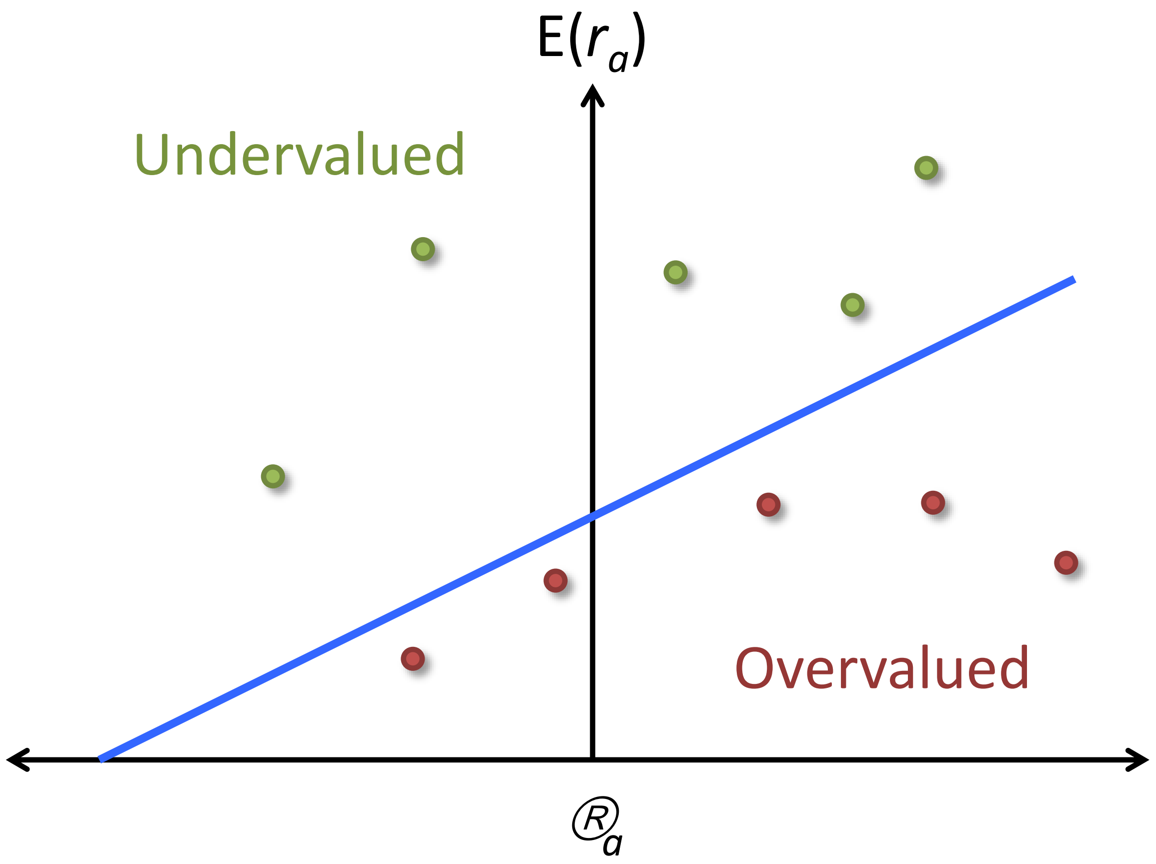
$$E(r_a) = r_f + [E(r_m) - r_f]\beta_a$$



Security market line

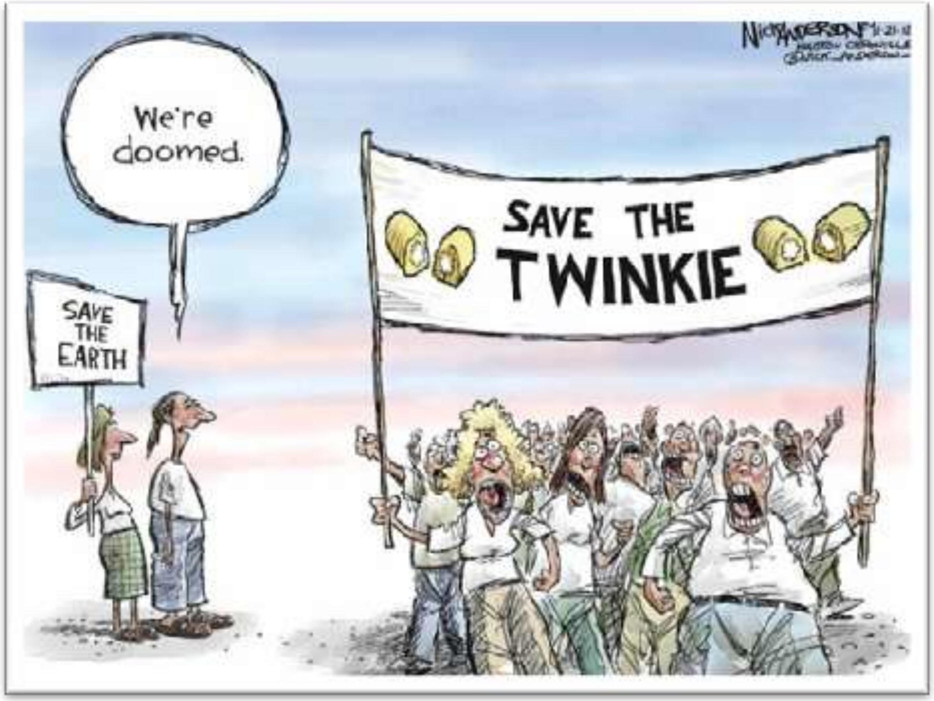


Security market line

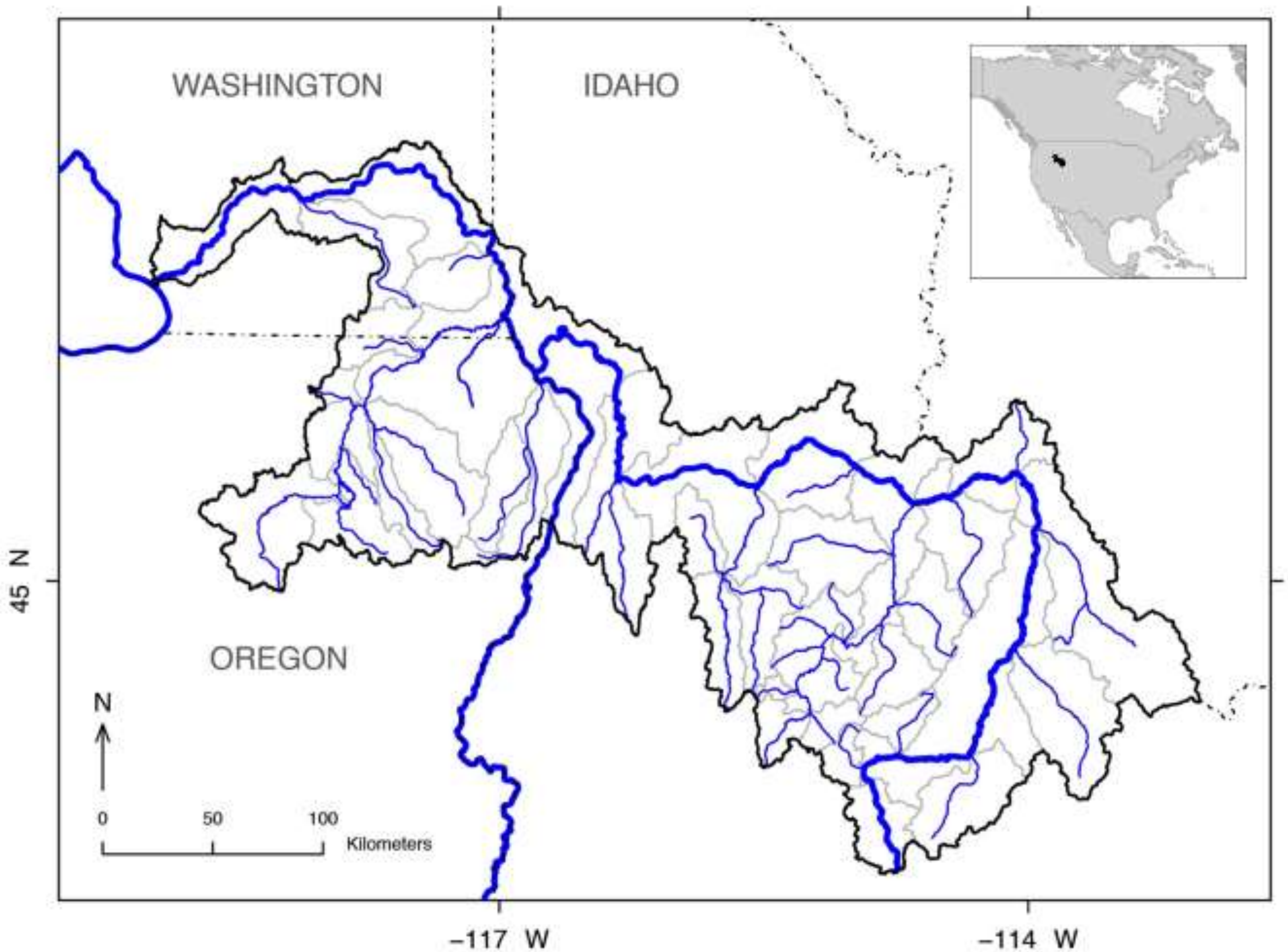


Ecological analogues

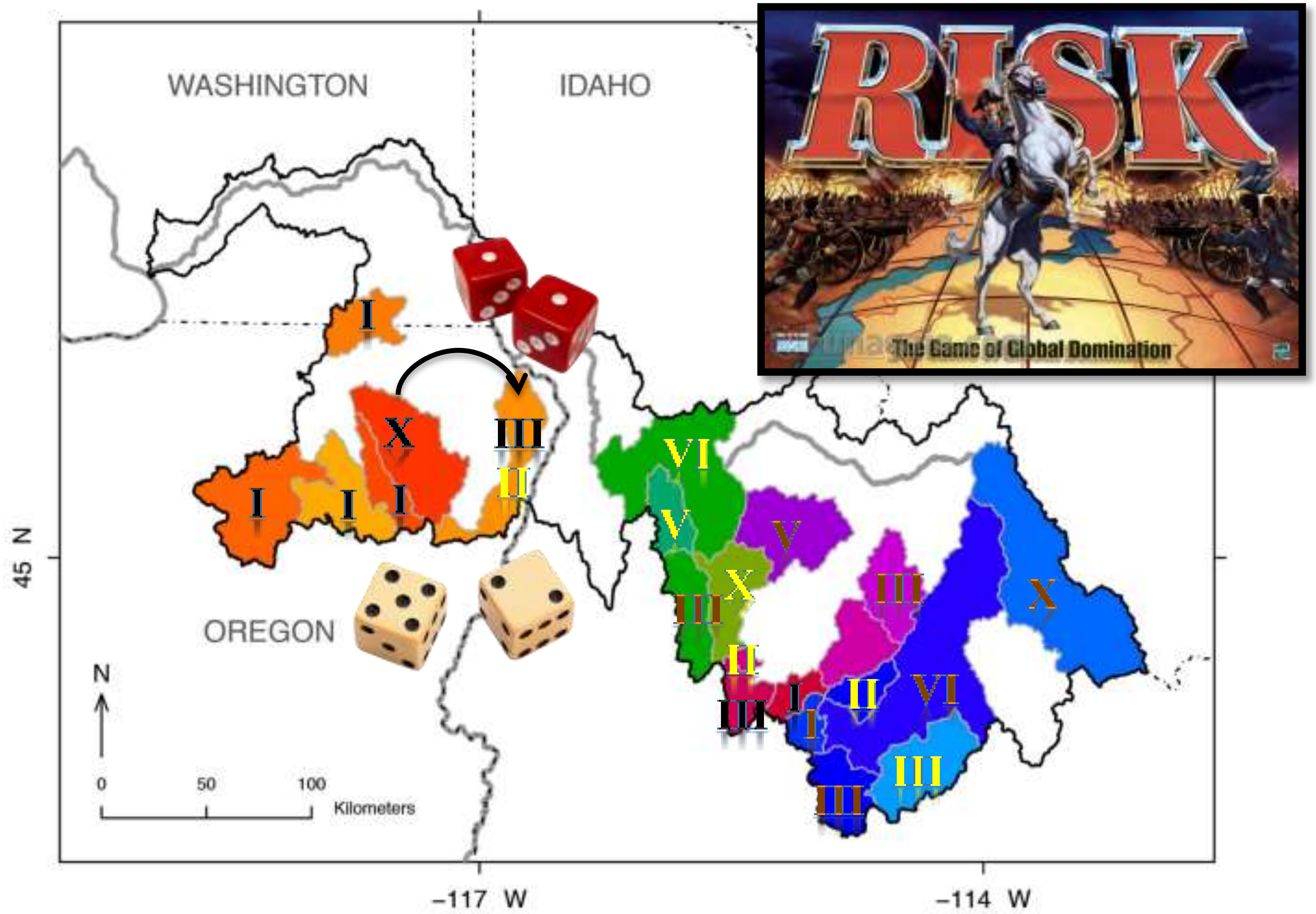
<u>Many</u> ecologists study "risk" & "returns" in a conservation context



Snake R spr/sum Chinook ESU



Snake R spr/sum Chinook ESU



Asset, market & risk-free indices

Assets

- ln[R/S]
- ln[S/ha]

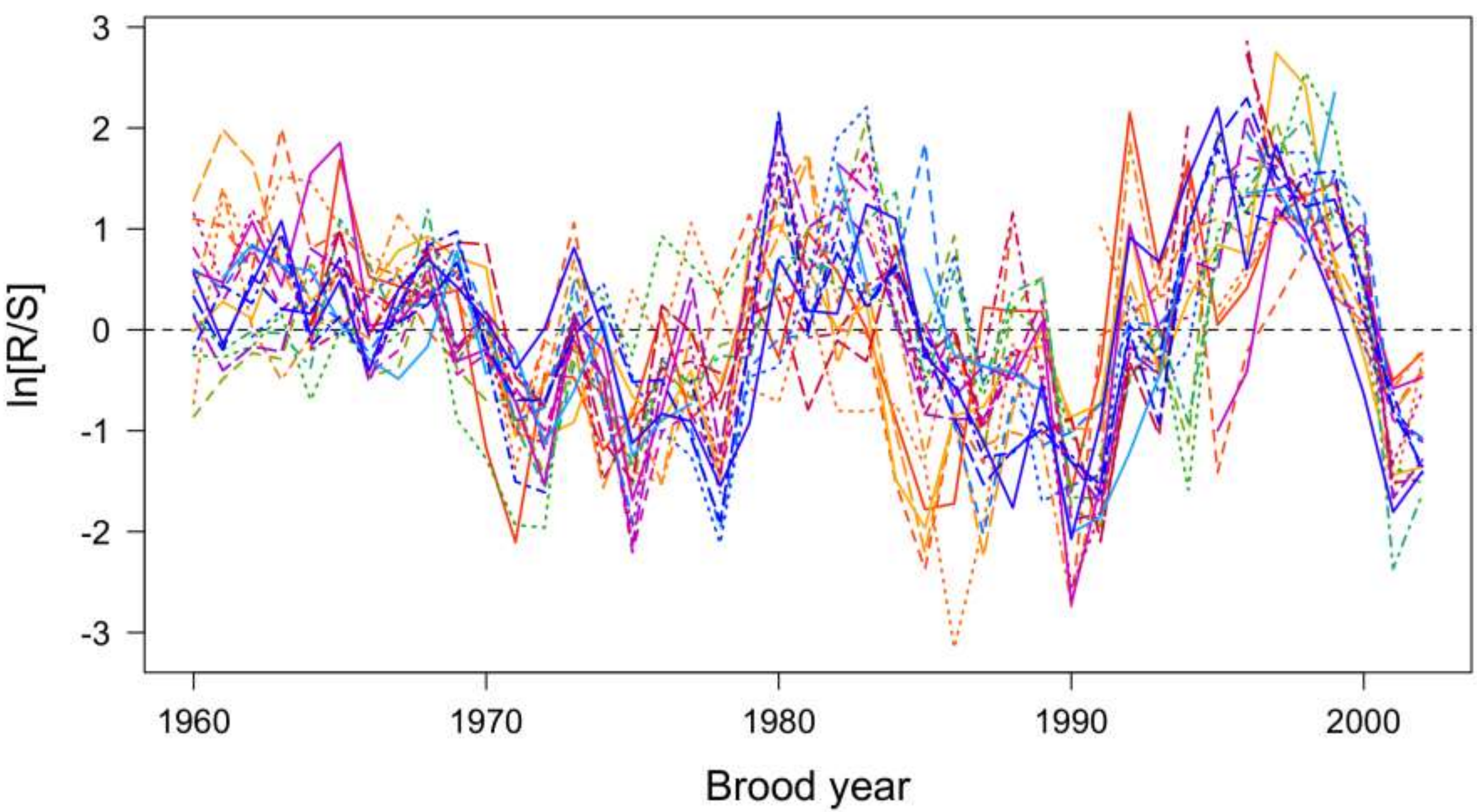
Market

- Pacific Decadal Oscillation (PDO) in brood yr + 2
- North Pacific Gyre Oscill (NPGO) in return yr - 1

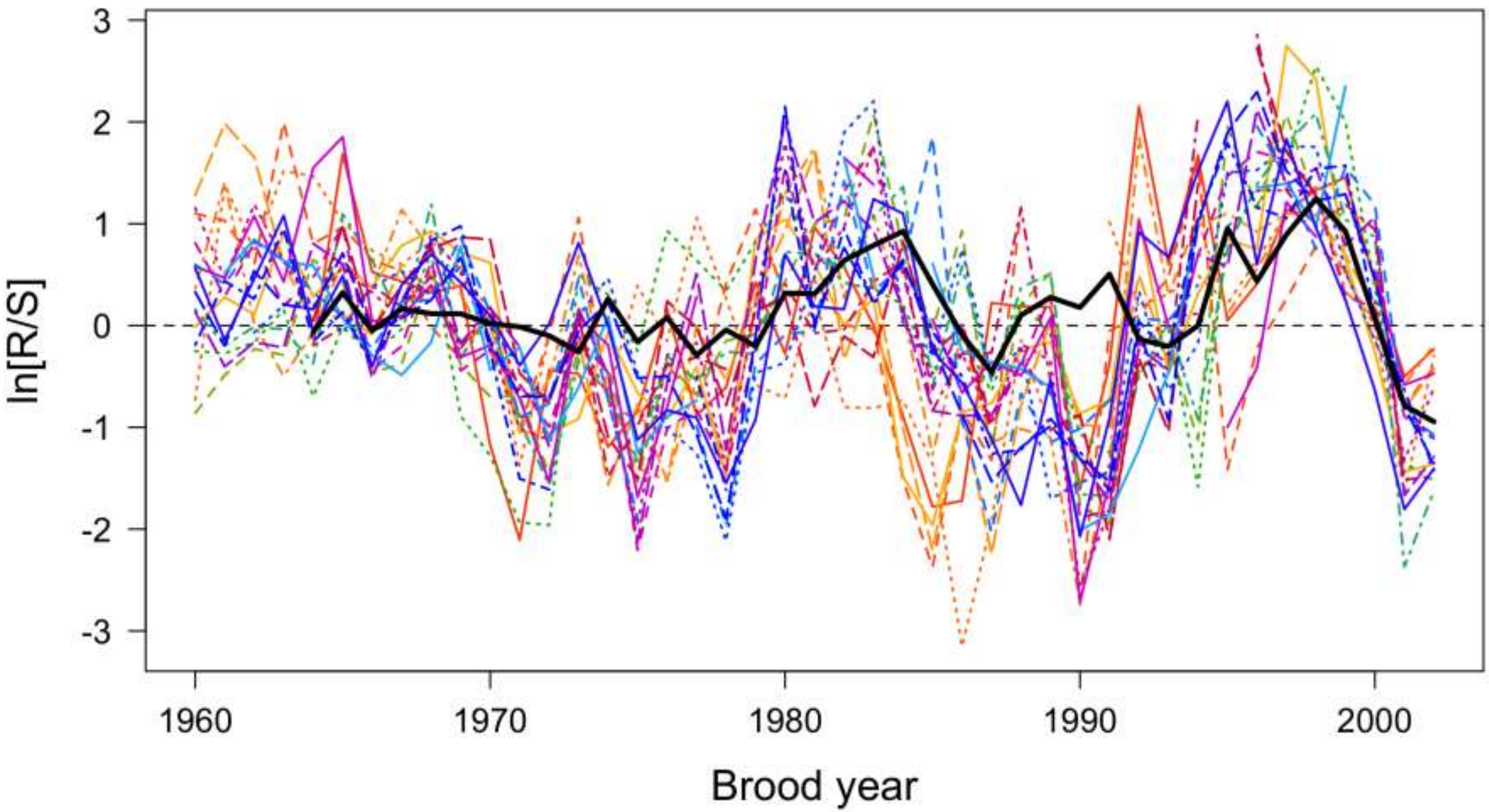
Risk-free

- Replacement (ln[R/S] = 0)
- ln[R/S] of John Day popn

Time series of “returns”



Time series of returns & risk-free



Fitting the market model

- In practice, the errors are often assumed to be Gaussian, and the model is solved via OLS
- This works well if the underlying relationship between asset & market is constant, but that rarely holds
- One option is to pass a moving window through the data, but window size affects accuracy & precision of β
- Better choice is to use a dynamic linear model (DLM)

Linear regression in matrix form

- Let's write the model

$$r_{a,t} = a_a + b_a r_{m,t} + v_{a,t} \qquad v_{a,t} \sim \mathrm{N}(0, s)$$

in matrix notation as

$$r_{a,t} = \mathbf{R}_{m,t} \theta_a + v_{a,t}$$

where

$$\mathbf{R}_{m,t} = \begin{pmatrix} 1 & R_{m,t} \end{pmatrix} \quad \& \quad \theta_a = \begin{pmatrix} \alpha_a \\ \beta_a \end{pmatrix}$$

Dynamic linear model

In a *dynamic* linear model, the regression parameters change over time, so we write

$$r_{a,t} = \mathbf{R}_{m,t}\theta_{a} + v_{a,t} \qquad (\textit{static})$$

as

$$r_{a,t} = \mathbf{R}_{m,t}\theta_{a,t} + v_{a,t} \qquad (\textit{dynamic})$$

Relationship between market & asset is unique at every *t*

Constraining a DLM

- Examination of the DLM reveals an apparent complication for parameter estimation

$$r_{a,t} = \mathbf{R}_{m,t} \theta_{a,t} + v_{a,t}$$

- With only 1 obs at each *t*, we could only hope to estimate 1 parameter (and no uncertainty)!

- To address this, we will constrain the regression parameters to be dependent from *t* to *t*+1

$$\theta_{a,t} = \mathbf{G}_t \theta_{a,t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \text{MVN}(\mathbf{0}, \mathbf{Q})$$

In practice, **G** is often time invariant (& set to **I**)

Summary of a market DLM

Observation equation

$$r_{a,t} = \mathbf{R}_{m,t}\theta_{a,t} + v_{a,t} \qquad v_t \sim \mathrm{N}(0, s)$$

Relates market index to the asset

State or "evolution" equation

$$\boldsymbol{\theta}_{a,t} = \mathbf{G}_t \boldsymbol{\theta}_{a,t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{Q})$$

Determines how parameters change over time

Forecasting with univariate DLM

- DLMs are often used in a forecasting context where we are interested in a prediction at time *t* conditioned on data up through time *t*-1
- Beginning with the distribution of $\boldsymbol{\theta}$ at time *t*-1 conditioned on the data through time *t*-1:

$$\boldsymbol{\theta}_{t-1} \mid y_{1:t-1} \sim \mathbf{MVN}\left(\boldsymbol{\pi}_{t-1}, \boldsymbol{\Lambda}_{t-1}\right)$$

- Then, the predictive distribution for θ_t given $y_{1:t-1}$ is:

$$\boldsymbol{\theta}_t \mid y_{1:t-1} \sim \mathbf{MVN}\left(\mathbf{G}_t \boldsymbol{\pi}_{t-1}, \mathbf{G}_t \boldsymbol{\Lambda}_{t-1} \mathbf{G}_t^\top + \mathbf{Q}\right)$$

Multivariate DLM

- Here we will examine multiple assets at once, so we need a multivariate (response) DLM

- First, the obs eqn

$$r_{a,t} = \mathbf{R}_{m,t}\theta_{a,t} + v_{a,t} \qquad v_t \sim \mathrm{N}\left(0, s\right)$$

becomes

$$\mathbf{R}_t = \left(\mathbf{R}_{m,t} \otimes \mathbf{I}_n\right)\theta_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{\Sigma})$$

Multivariate DLM – obs eqn

$$\mathbf{R}_t = \left(\mathbf{R}_{m,t} \otimes \mathbf{I}_n\right)\theta_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim \mathrm{MVN}(\mathbf{0}, \boldsymbol{\Sigma})$$

$$\begin{bmatrix} r_{1,t} \\ \vdots \\ r_{n,t} \end{bmatrix} = \begin{bmatrix} 1 & R_{m,t} \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multivariate DLM – obs eqn

$$\mathbf{R}_t = \left(\mathbf{R}_{m,t} \otimes \mathbf{I}_n\right)\theta_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim \mathrm{MVN}(\mathbf{0}, \boldsymbol{\Sigma})$$

$$\begin{bmatrix} r_{1,t} \\ \vdots \\ r_{n,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & r_{m,t} & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & 1 & 0 & 0 & r_{m,t} \end{bmatrix} \begin{bmatrix} \alpha_{1,t} \\ \vdots \\ \alpha_{n,t} \\ \beta_{1,t} \\ \vdots \\ \beta_{n,t} \end{bmatrix} + \begin{bmatrix} v_{1,t} \\ \vdots \\ v_{n,t} \end{bmatrix}$$

Multivariate DLM – obs eqn

$$\mathbf{R}_t = \left(\mathbf{R}_{m,t} \otimes \mathbf{I}_n\right)\theta_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim \mathrm{MVN}(\mathbf{0}, \underline{\mathbf{\Sigma}})$$

$$\underline{\mathbf{\Sigma} = \begin{bmatrix} \sigma_1 & \gamma_{21} & \cdots & \gamma_{n1} \\ \gamma_{12} & \sigma_2 & & \gamma_{n2} \\ \vdots & & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \sigma_n \end{bmatrix}} \qquad \underline{\mathbf{\Sigma} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n \end{bmatrix}}$$

Multivariate DLM – evolution eqn

- The evolution eqn

$$\theta_{a,t} = \mathbf{G}_t \theta_{a,t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{Q})$$

becomes

$$\theta_t = (\mathbf{G}_t \otimes \mathbf{I}_n) \theta_{t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{Q})$$

$$\mathbf{G}_t = \mathbf{I}_2 \text{ Þ } \mathbf{G}_t \text{ Ä } \mathbf{I}_n = \mathbf{I}_{2n}$$

$$\theta_t = \theta_{t-1} + \mathbf{w}_t$$

Multivariate DLM – evolution eqn

$$\theta_t = \theta_{t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{Q})$$

$$\begin{bmatrix} a_{1,t} \\ \vdots \\ a_{n,t} \\ b_{1,t} \\ \vdots \\ b_{n,t} \end{bmatrix} = \begin{bmatrix} a_{1,t-1} \\ \vdots \\ a_{n,t-1} \\ b_{1,t-1} \\ \vdots \\ b_{n,t-1} \end{bmatrix} + \begin{bmatrix} w_{1,t}^{(a)} \\ \vdots \\ w_{n,t}^{(a)} \\ w_{1,t}^{(b)} \\ \vdots \\ w_{n,t}^{(b)} \end{bmatrix}$$

Multivariate DLM – evolution eqn

$$\theta_t = \theta_{t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \text{MVN}(\mathbf{0}, \underline{\mathbf{Q}})$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{(a)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}^{(b)} \end{bmatrix} \qquad \mathbf{Q}^{(\times)} = \begin{bmatrix} q_1^{(\times)} & c_{21}^{(\times)} & \cdots & c_{n1}^{(\times)} \\ c_{12}^{(\times)} & q_2^{(\times)} & & c_{n2}^{(\times)} \\ \vdots & & \ddots & \vdots \\ c_{1n}^{(\times)} & c_{2n}^{(\times)} & \cdots & q_n^{(\times)} \end{bmatrix}$$

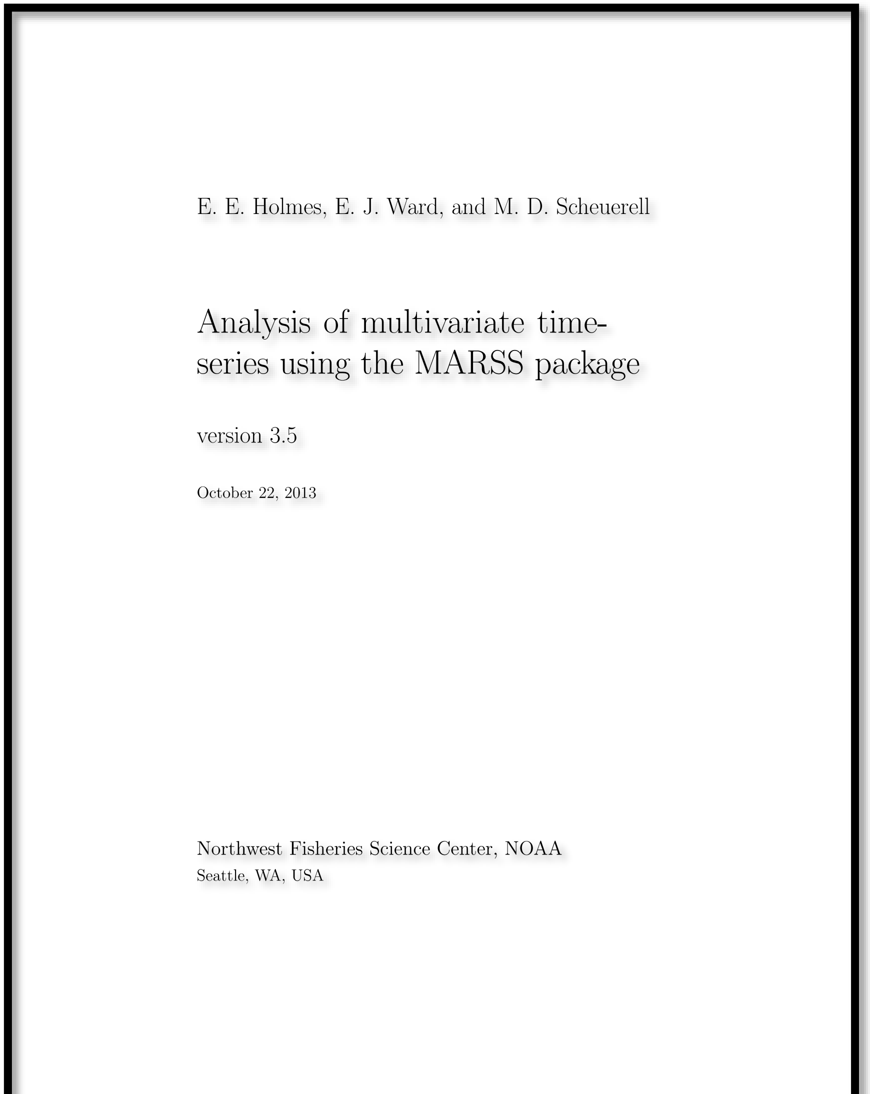
Multivariate DLM – evolution eqn

$$\theta_t = \theta_{t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \underline{\mathbf{Q}})$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1^{(a)} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Q}_k^{(a)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_1^{(b)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{Q}_k^{(b)} \end{bmatrix}$$

For $k < n$ "groups"

Fitting the models


E. E. Holmes, E. J. Ward, and M. D. Scheuerell

Analysis of multivariate time-series using the MARSS package

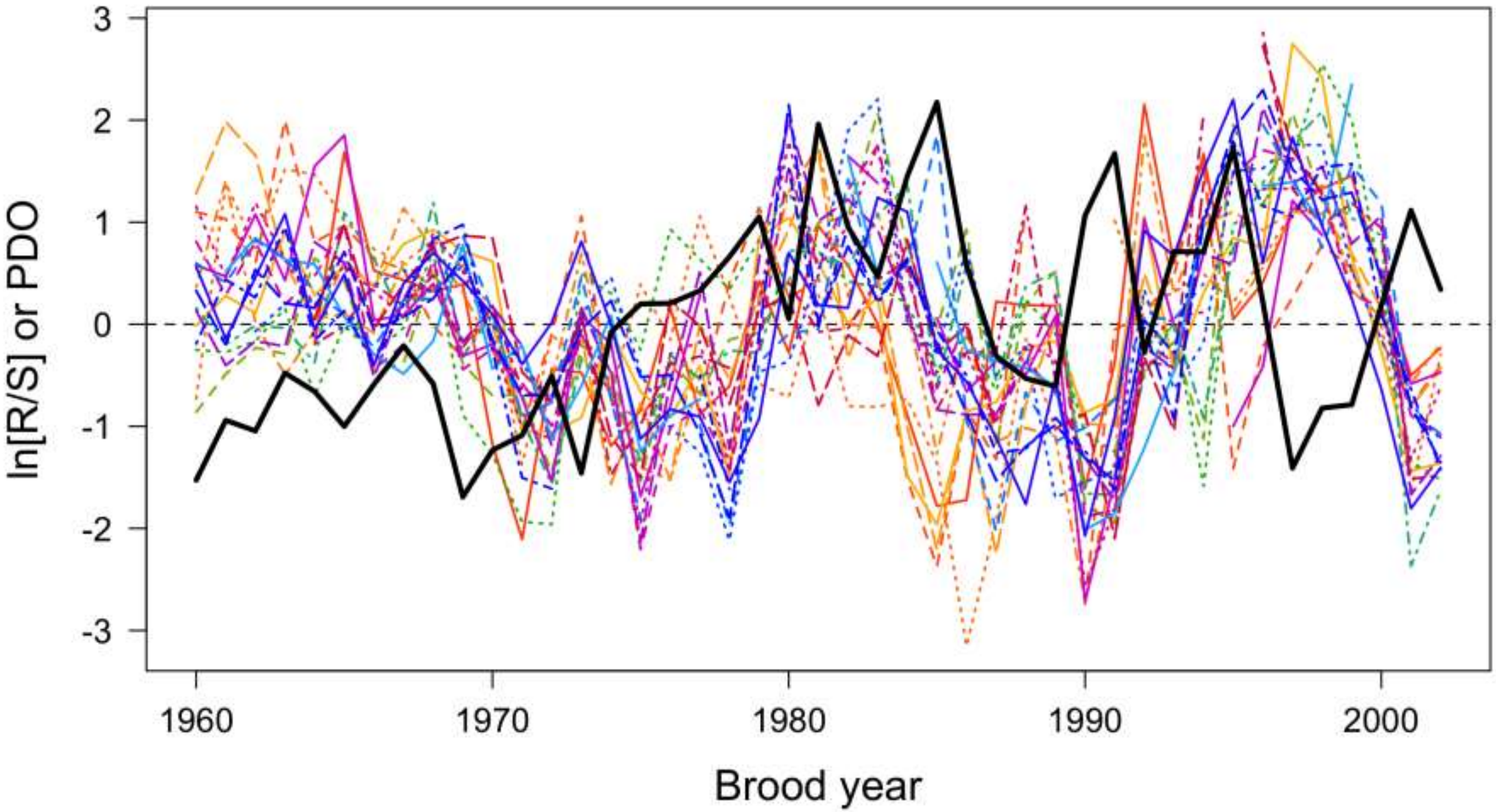
version 3.5

October 22, 2013

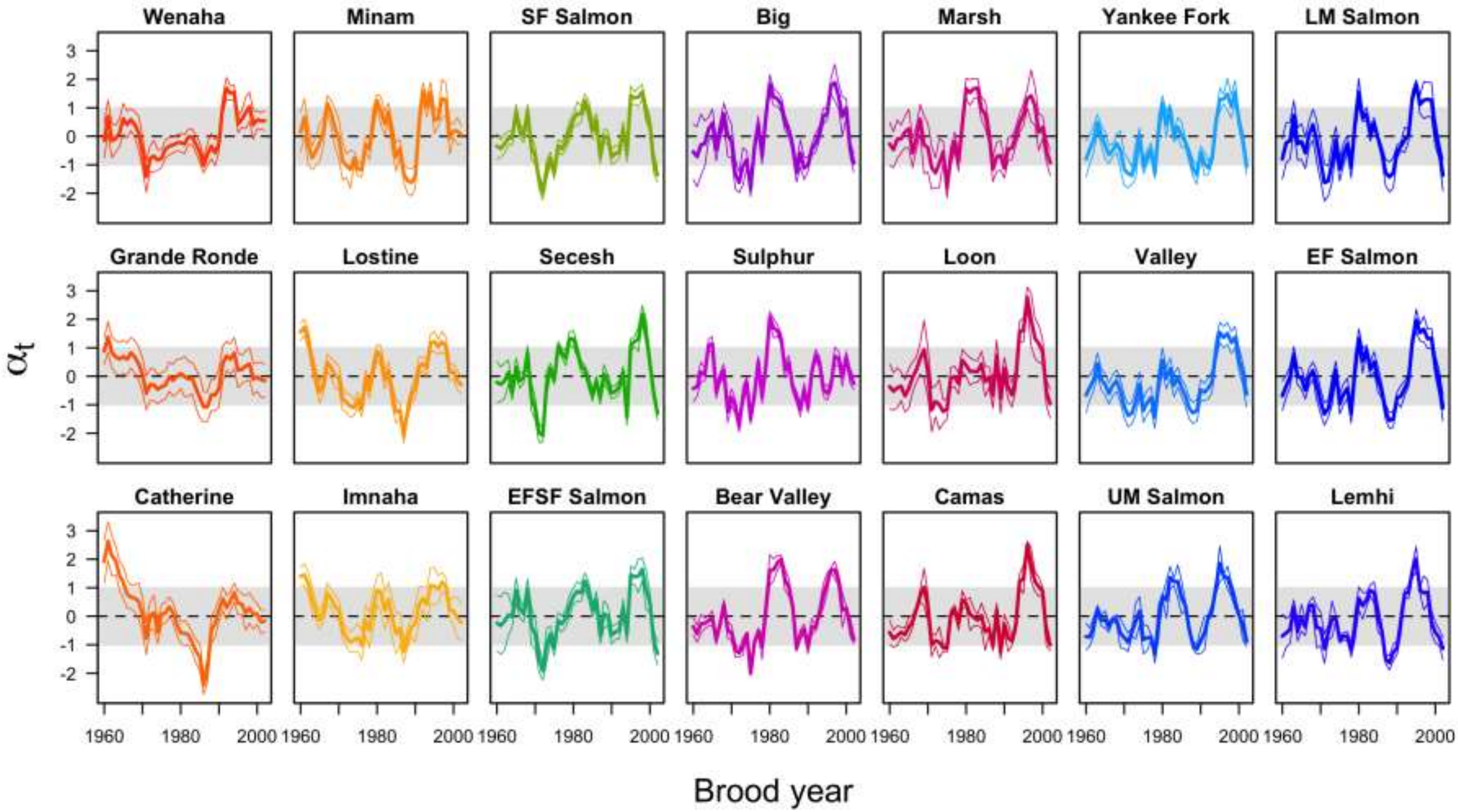
Northwest Fisheries Science Center, NOAA
Seattle, WA, USA

- Used the MARSS pkg in R
- Likelihood-based framework
- (Lots of other applications too)

Time series of “returns” & PDO

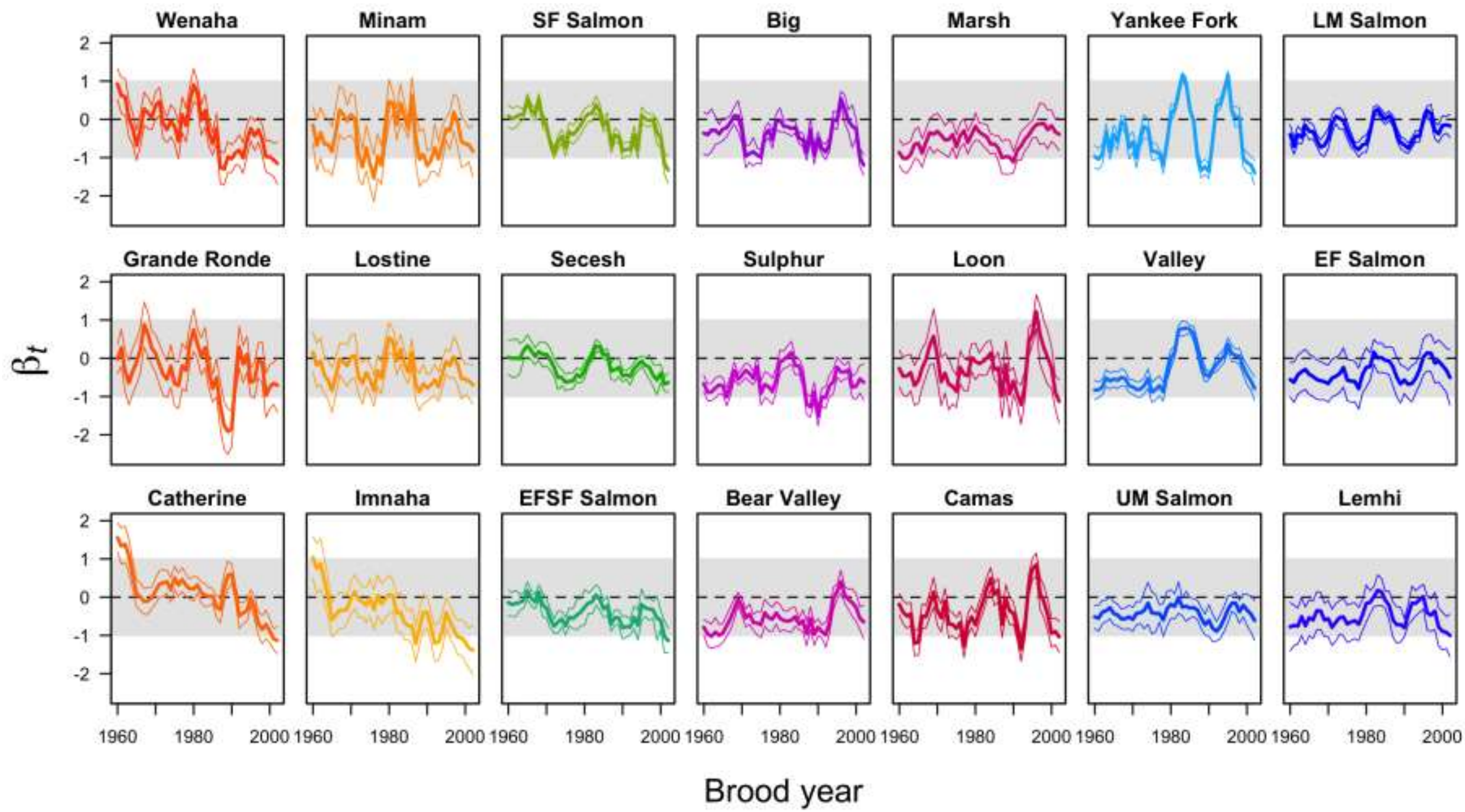


Results – time series of alphas



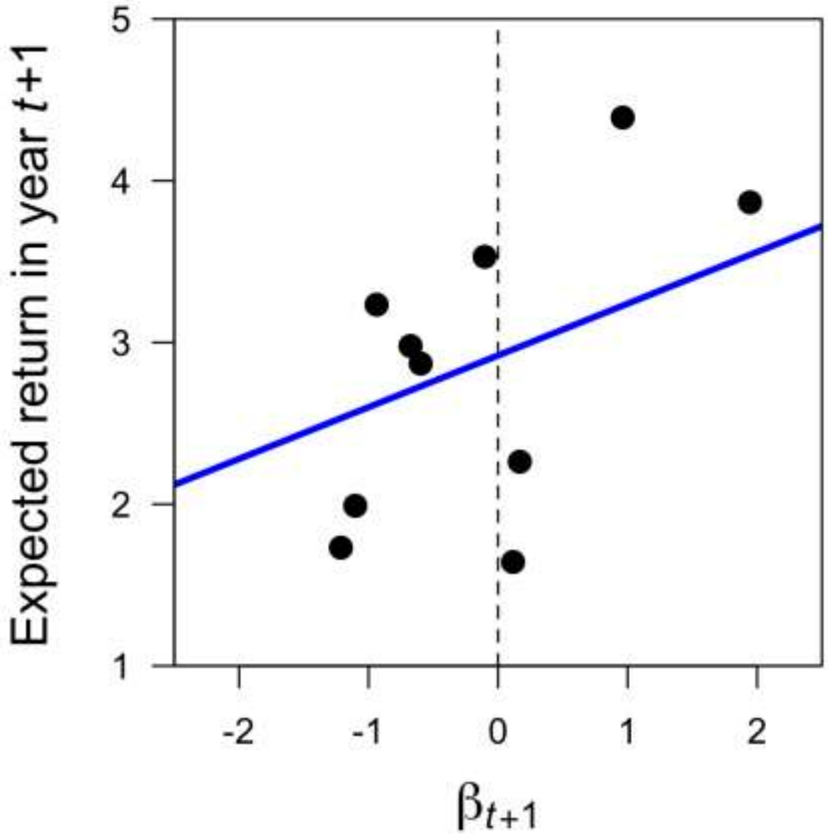
Market = PDO

Results – time series of betas

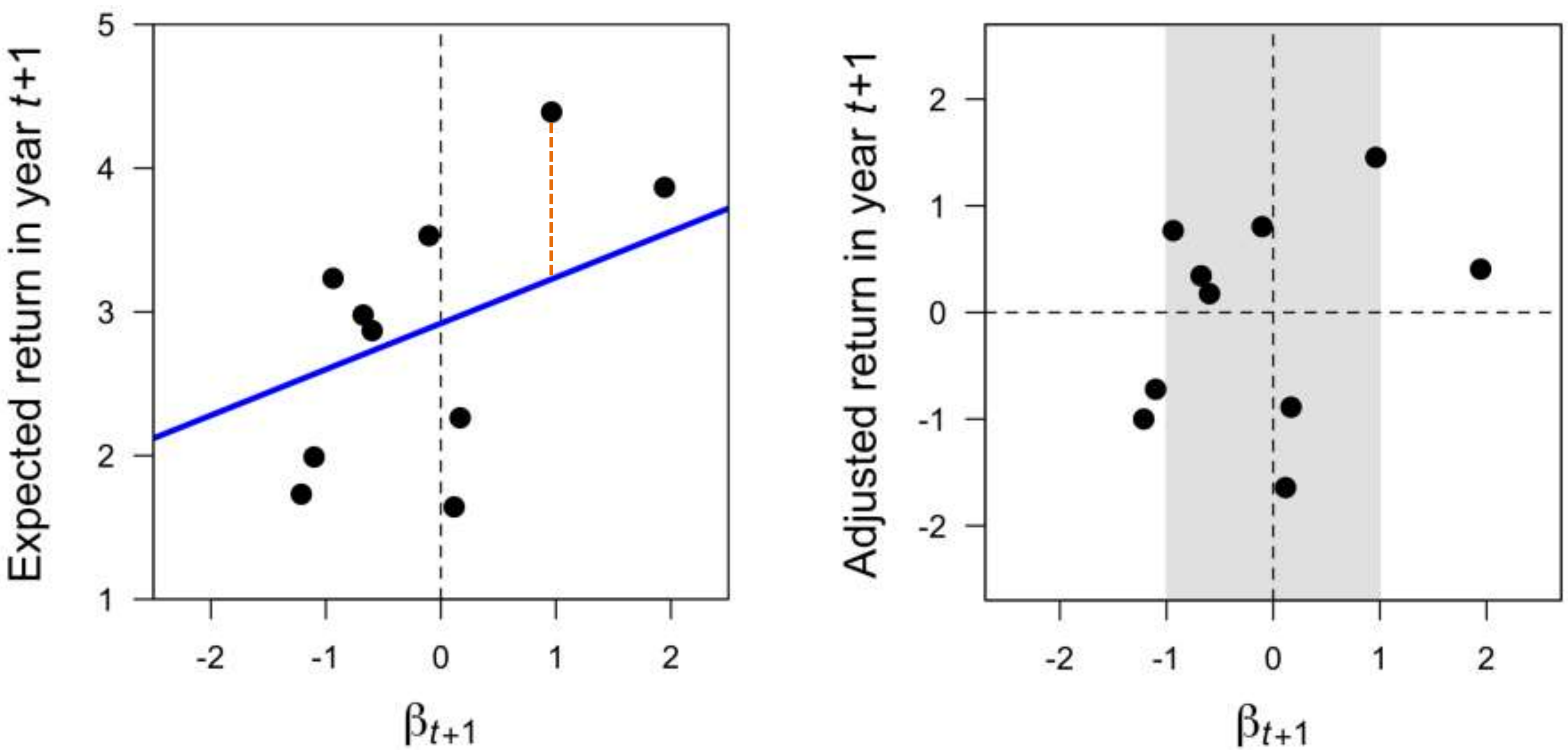


Market = PDO

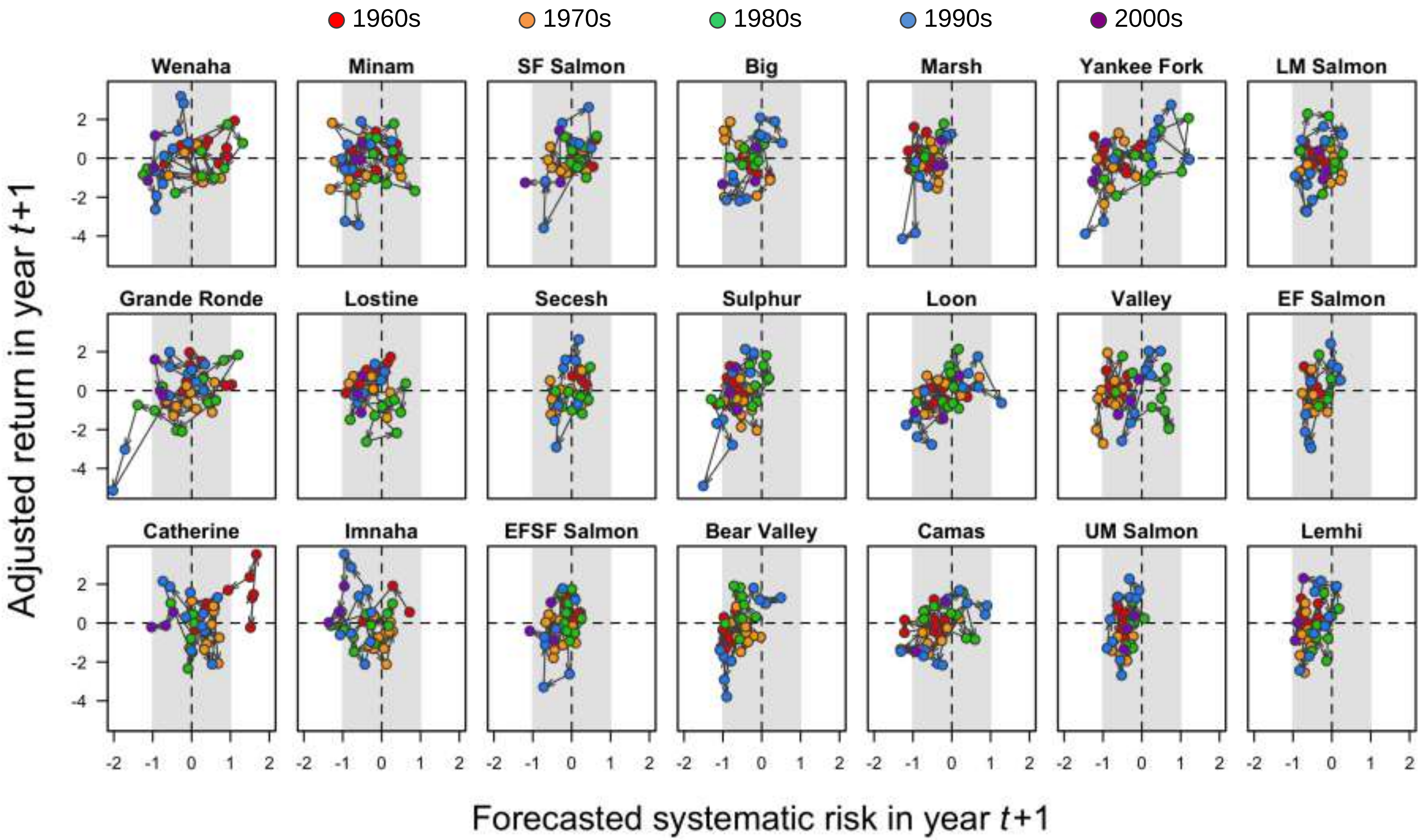
Security market line



Security market line

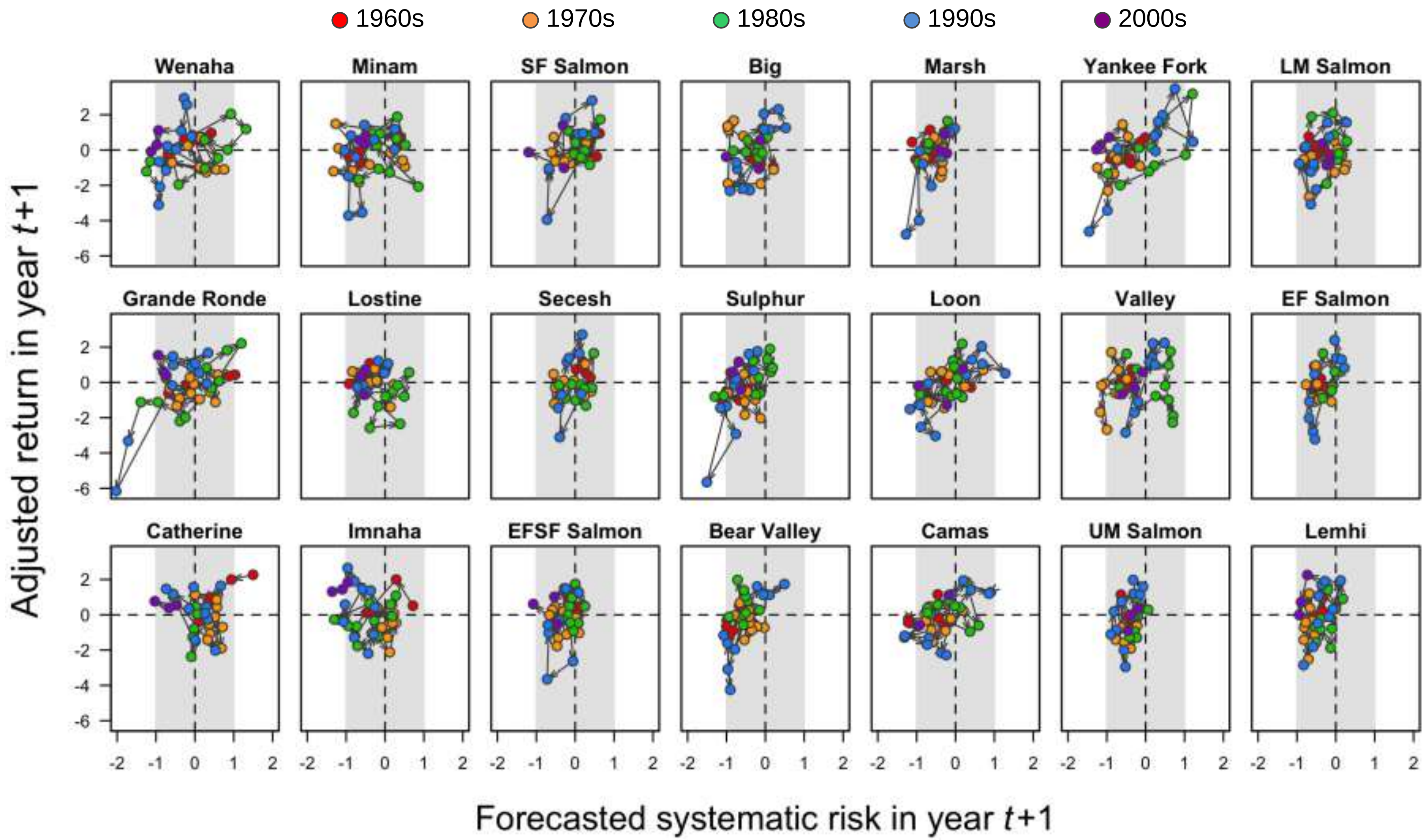


Results – CAPM



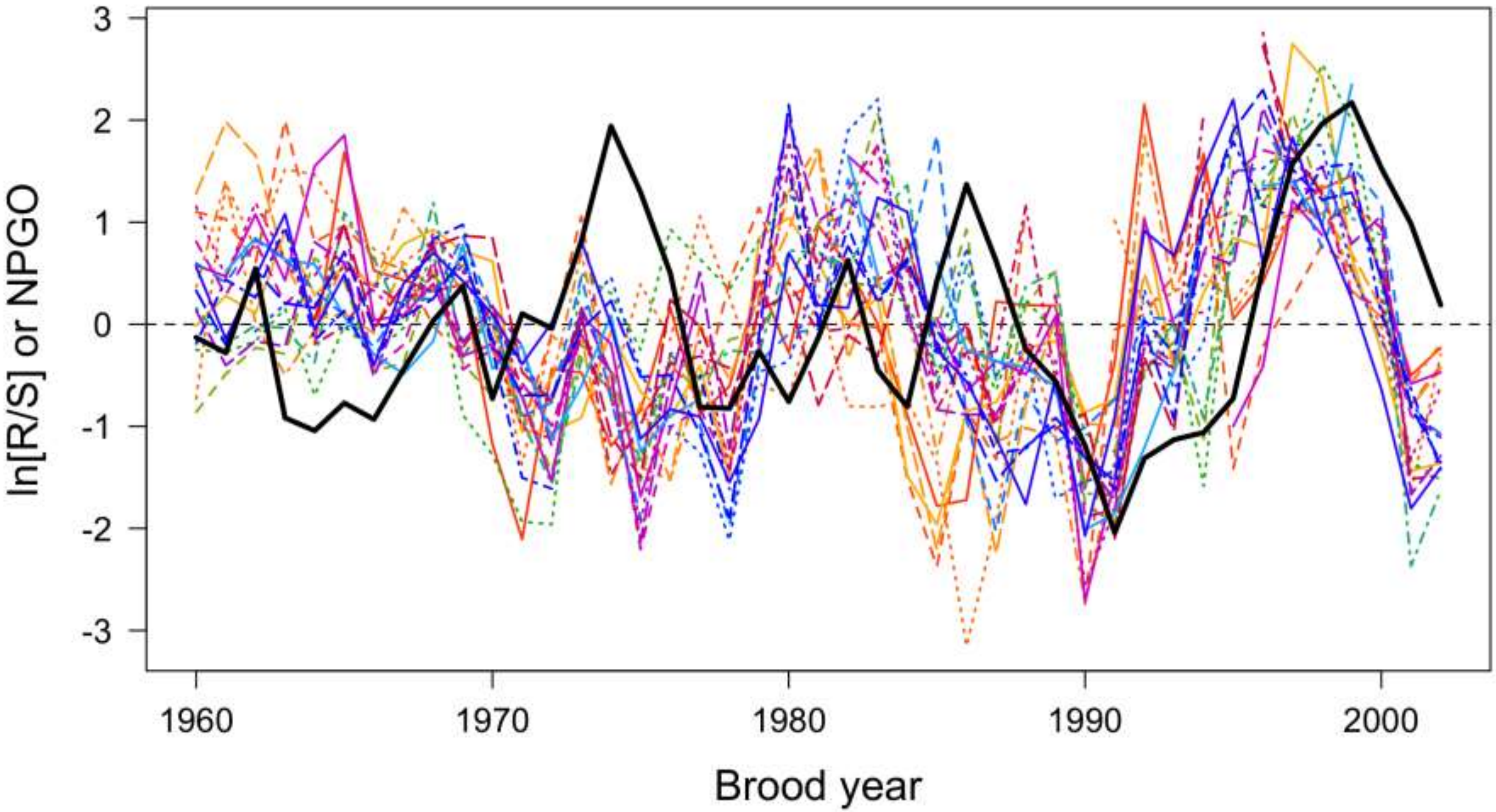
Market = PDO; *Risk-free* = 0

Results – CAPM

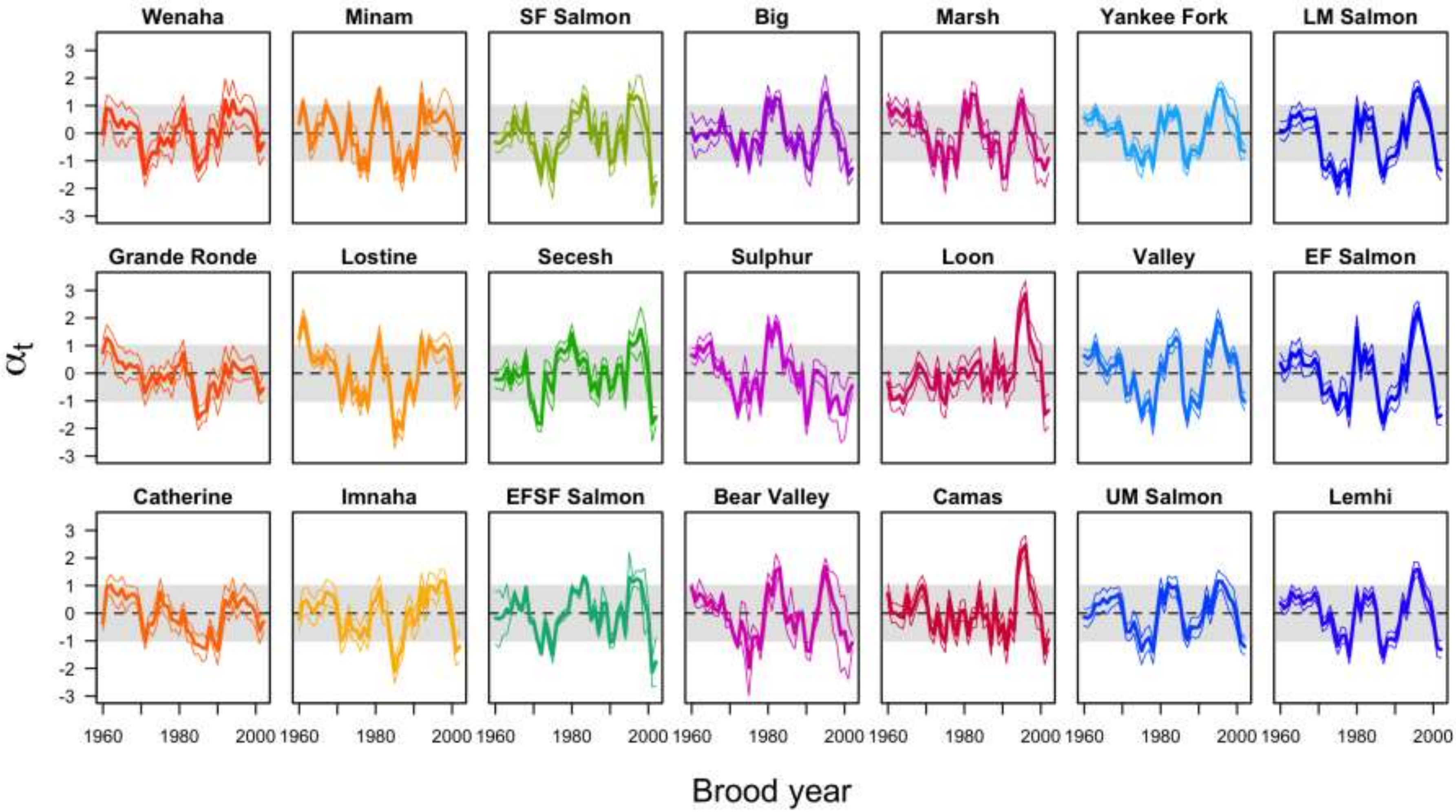


Market = PDO; *Risk-free* = John Day

Time series of returns & NPGO

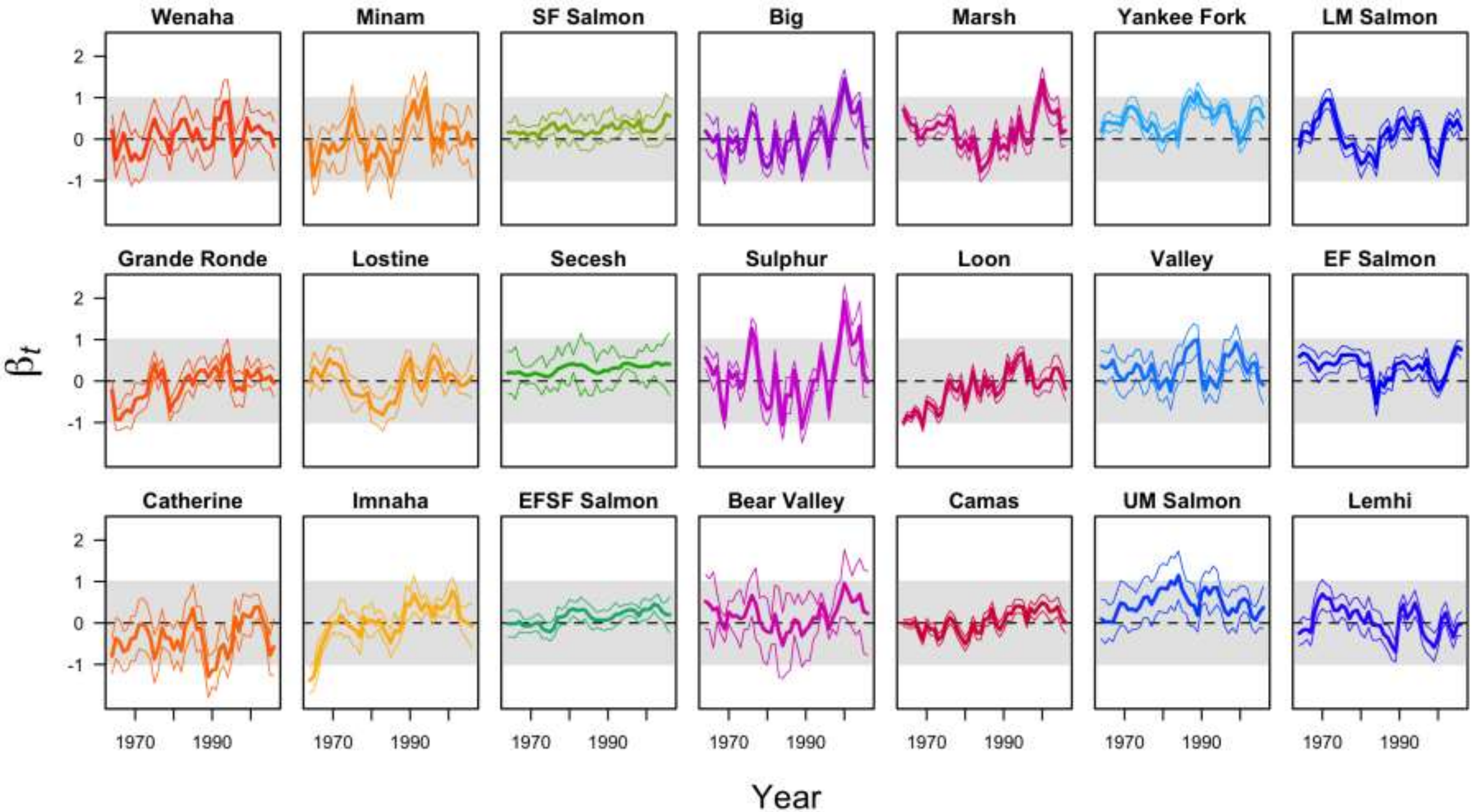


Results – time series of alphas



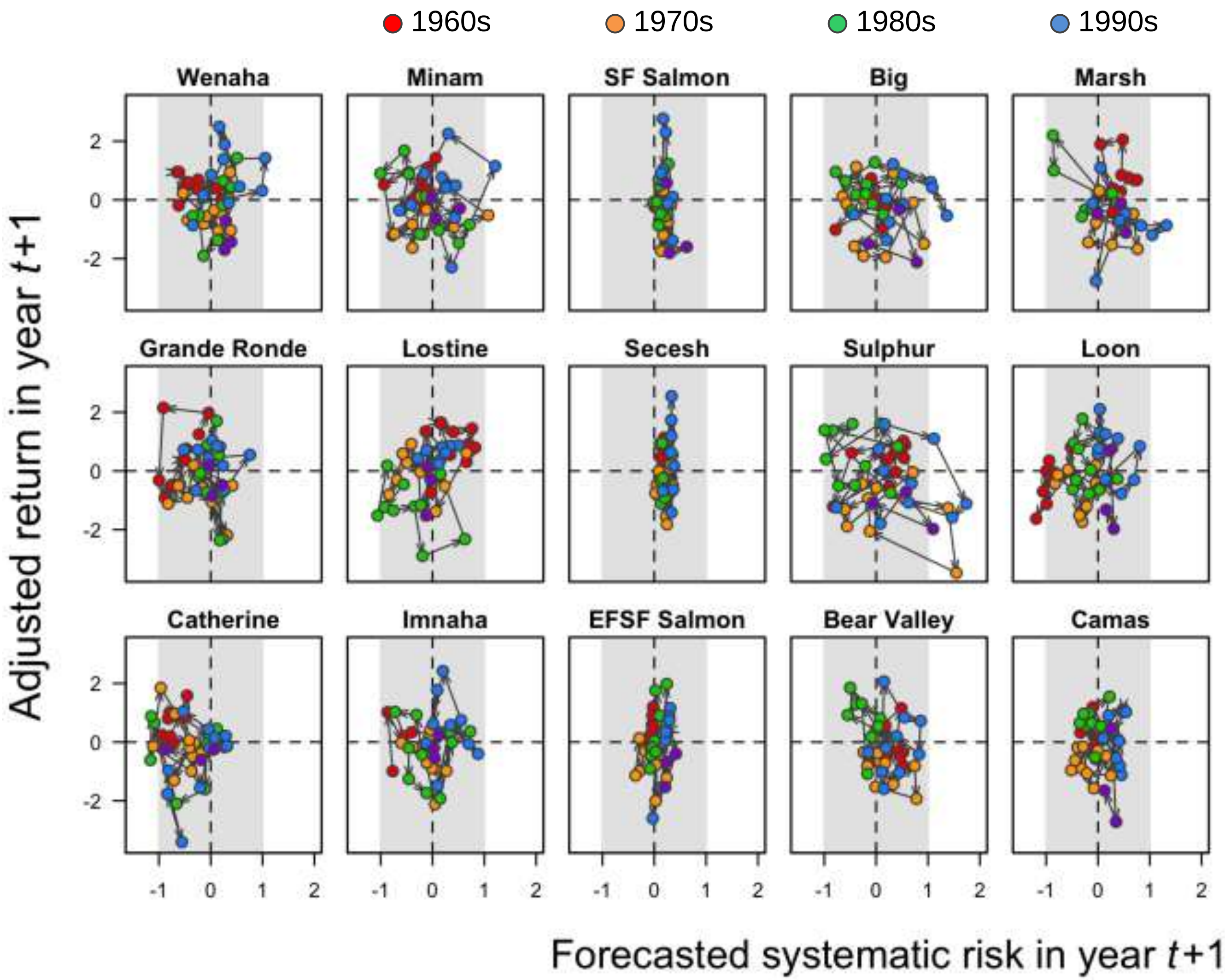
Market = NPGO

Results – time series of betas



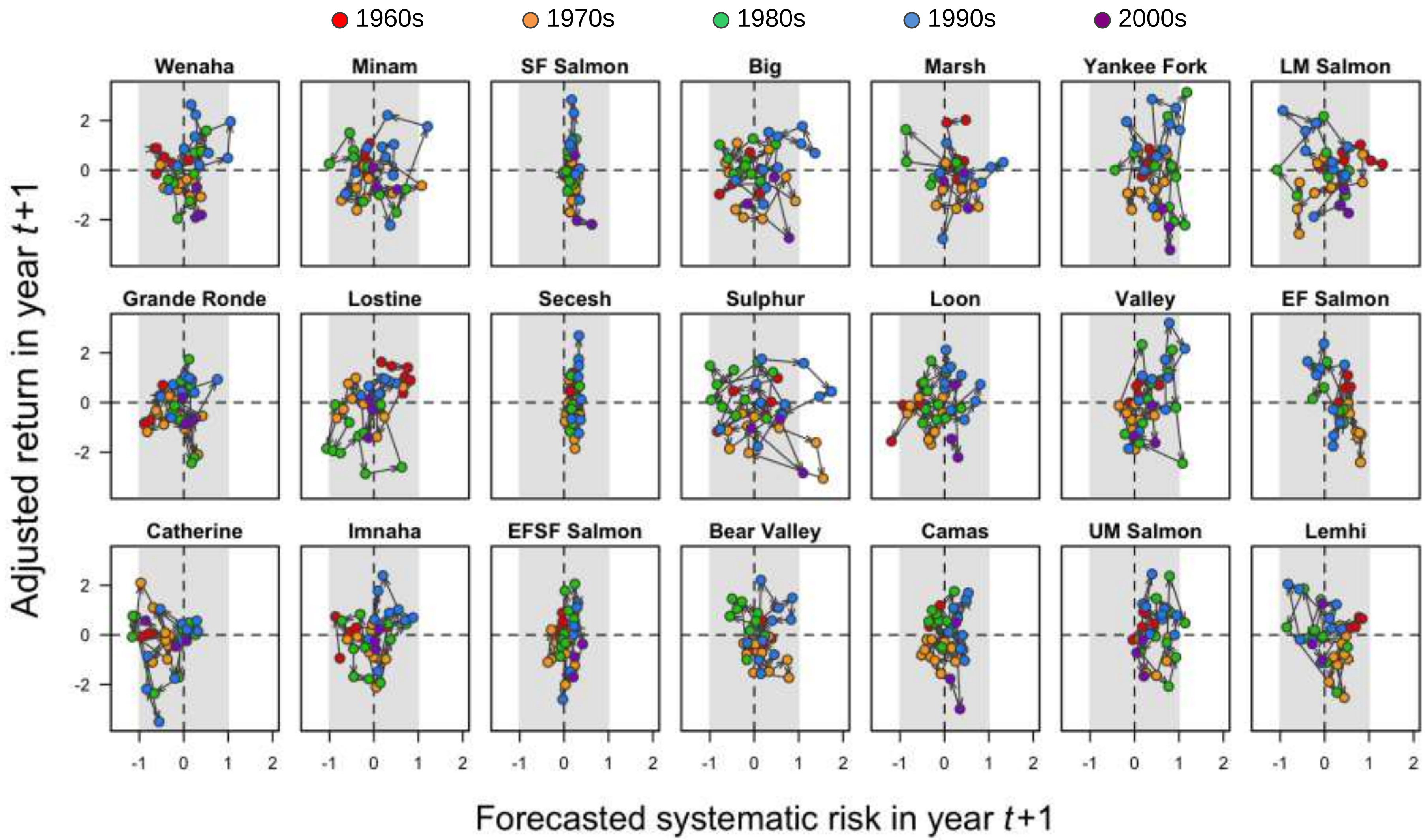
Market = NPGO

Results – CAPM



Market = NPGO; *Risk-free* = 0

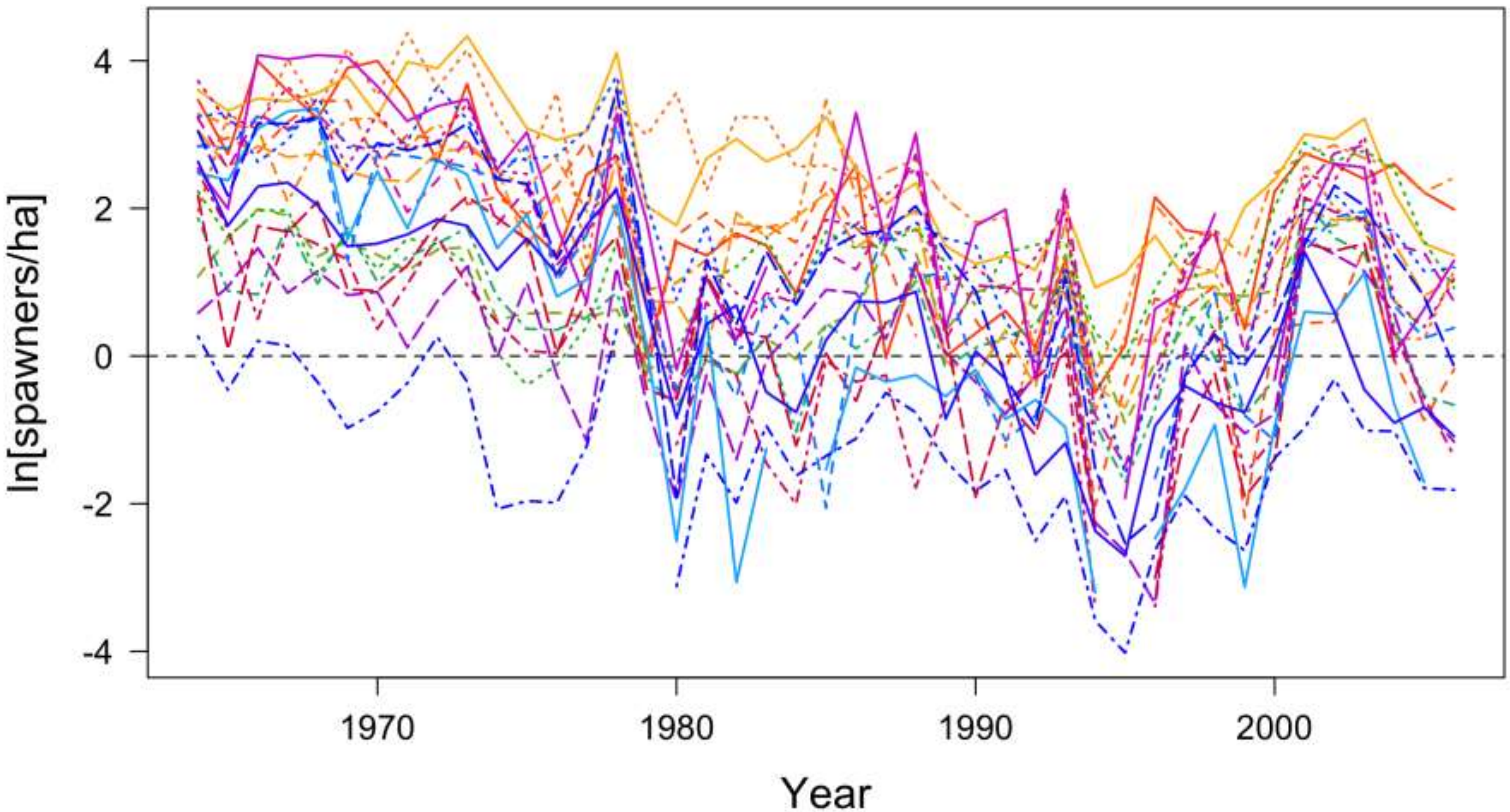
Results – CAPM



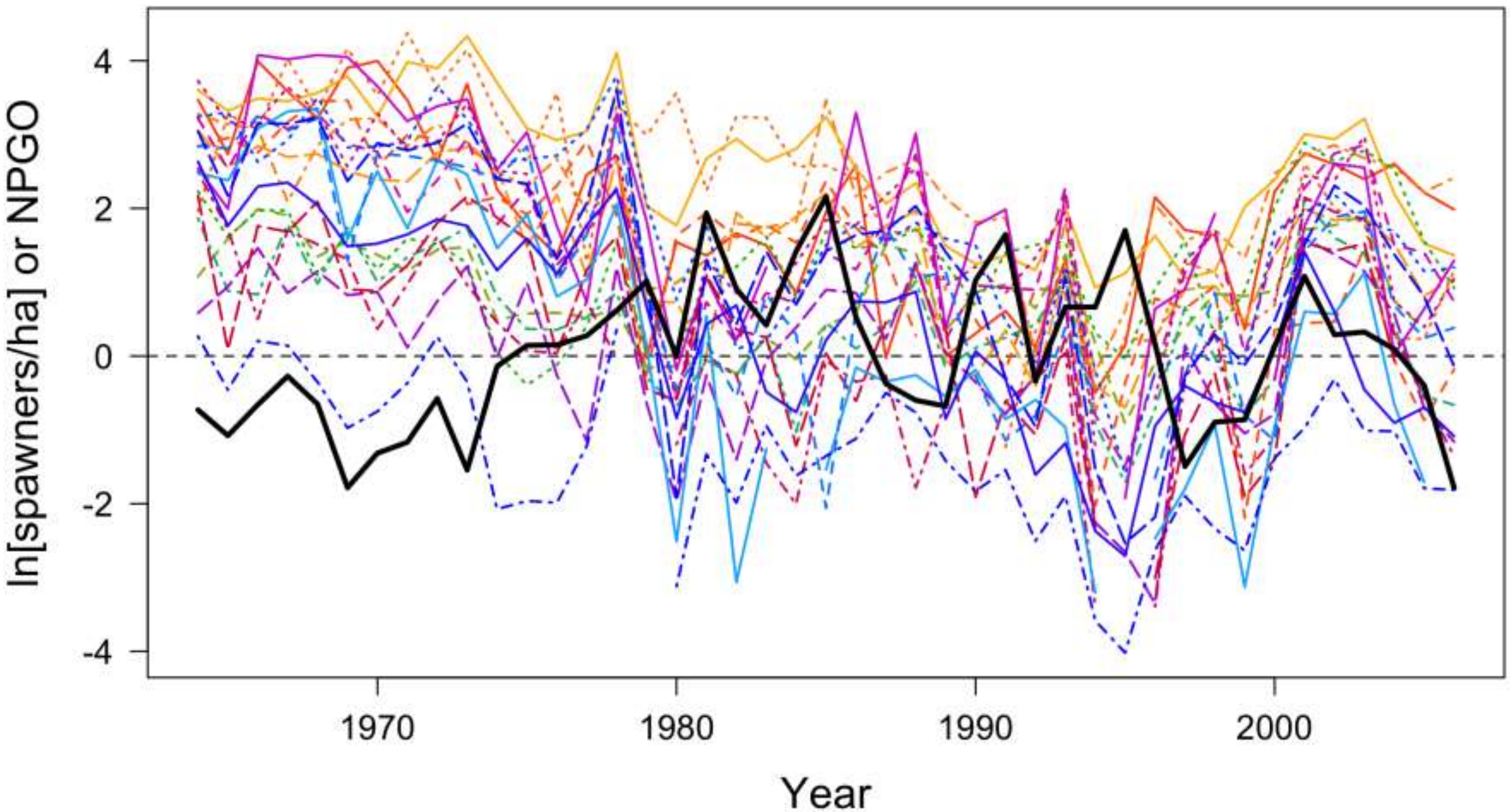
Market = NPGO; *Risk-free* = John Day

Photo by Jack Molan

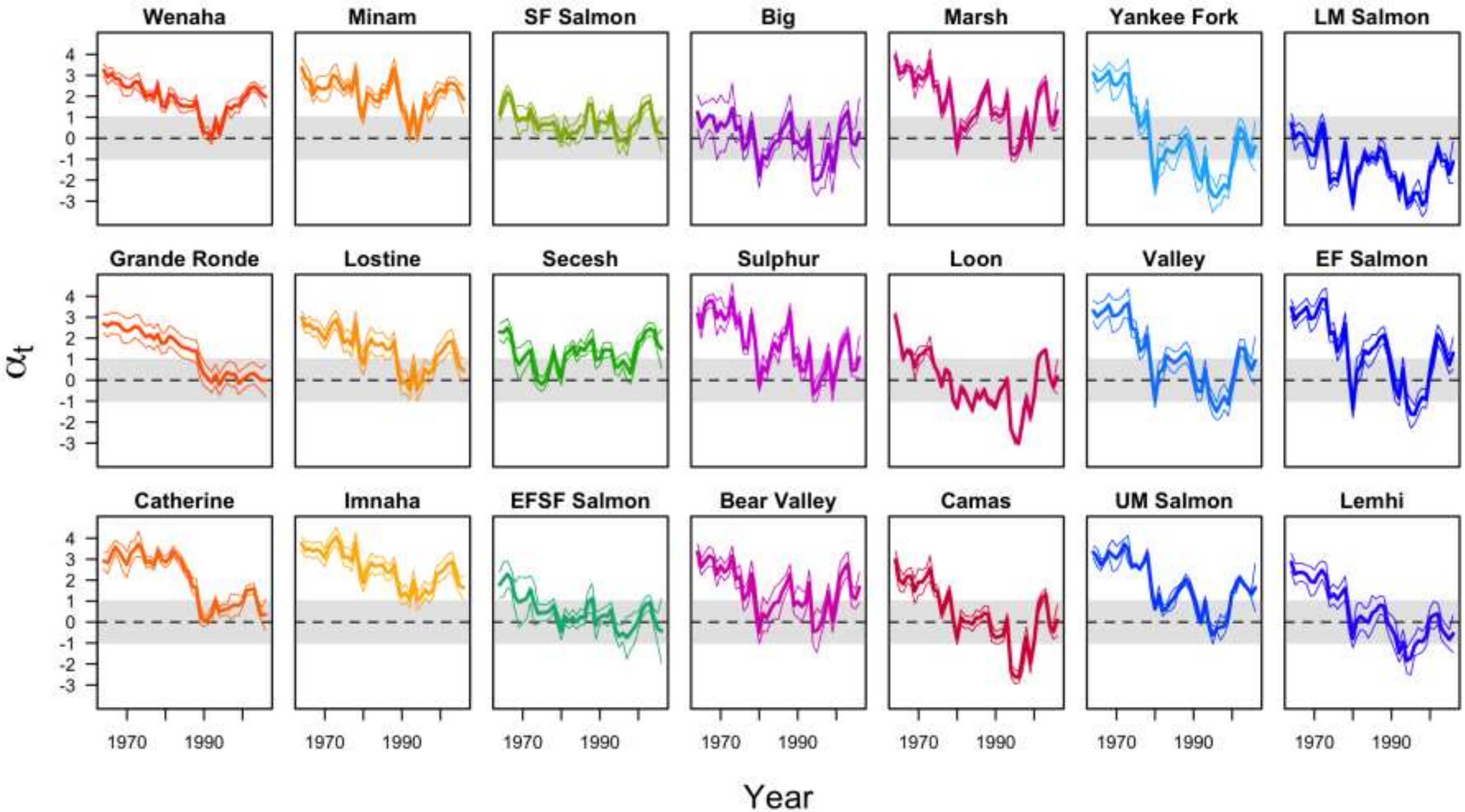
Time series of returns (Density)



Time series of returns & PDO

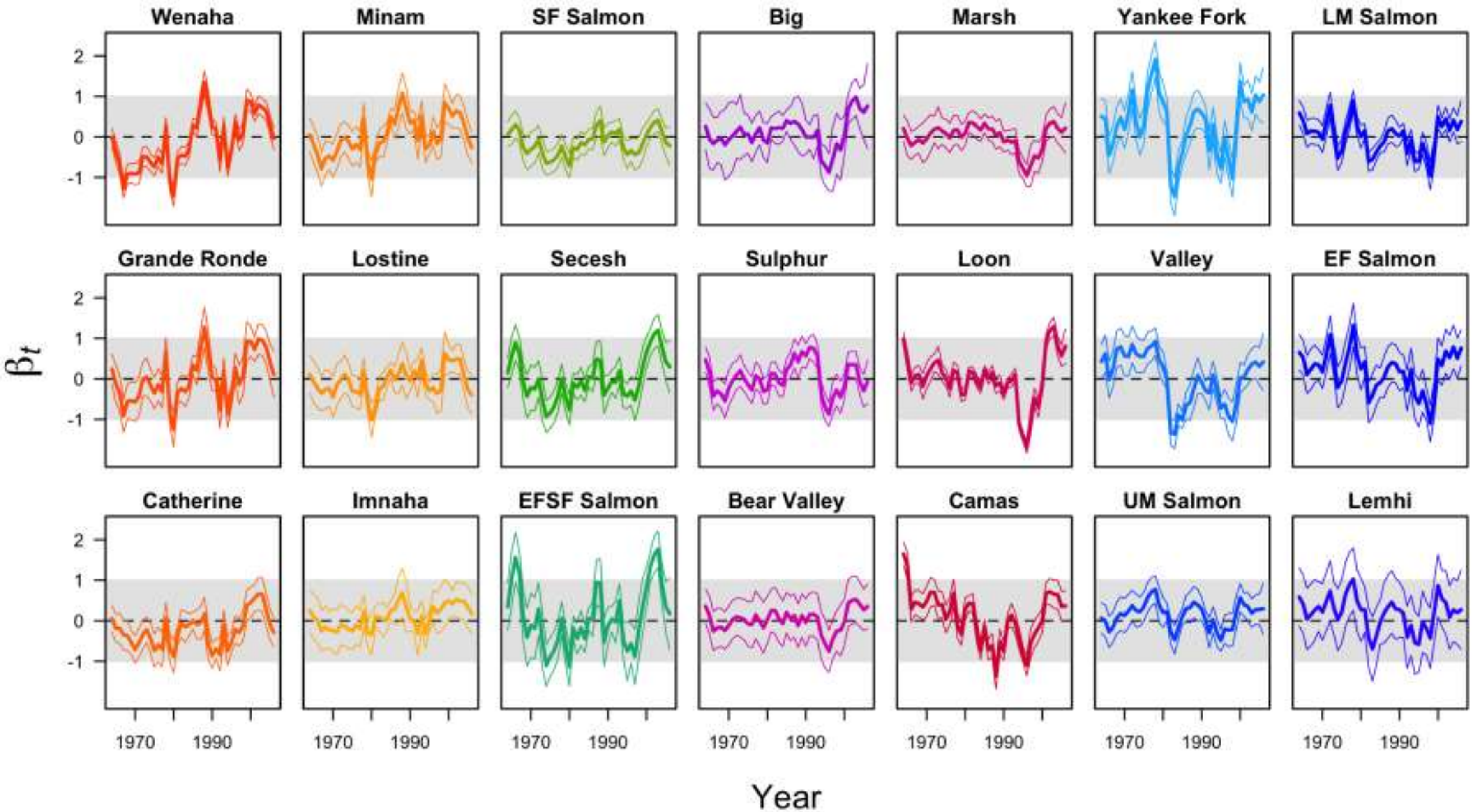


Results – time series of alphas



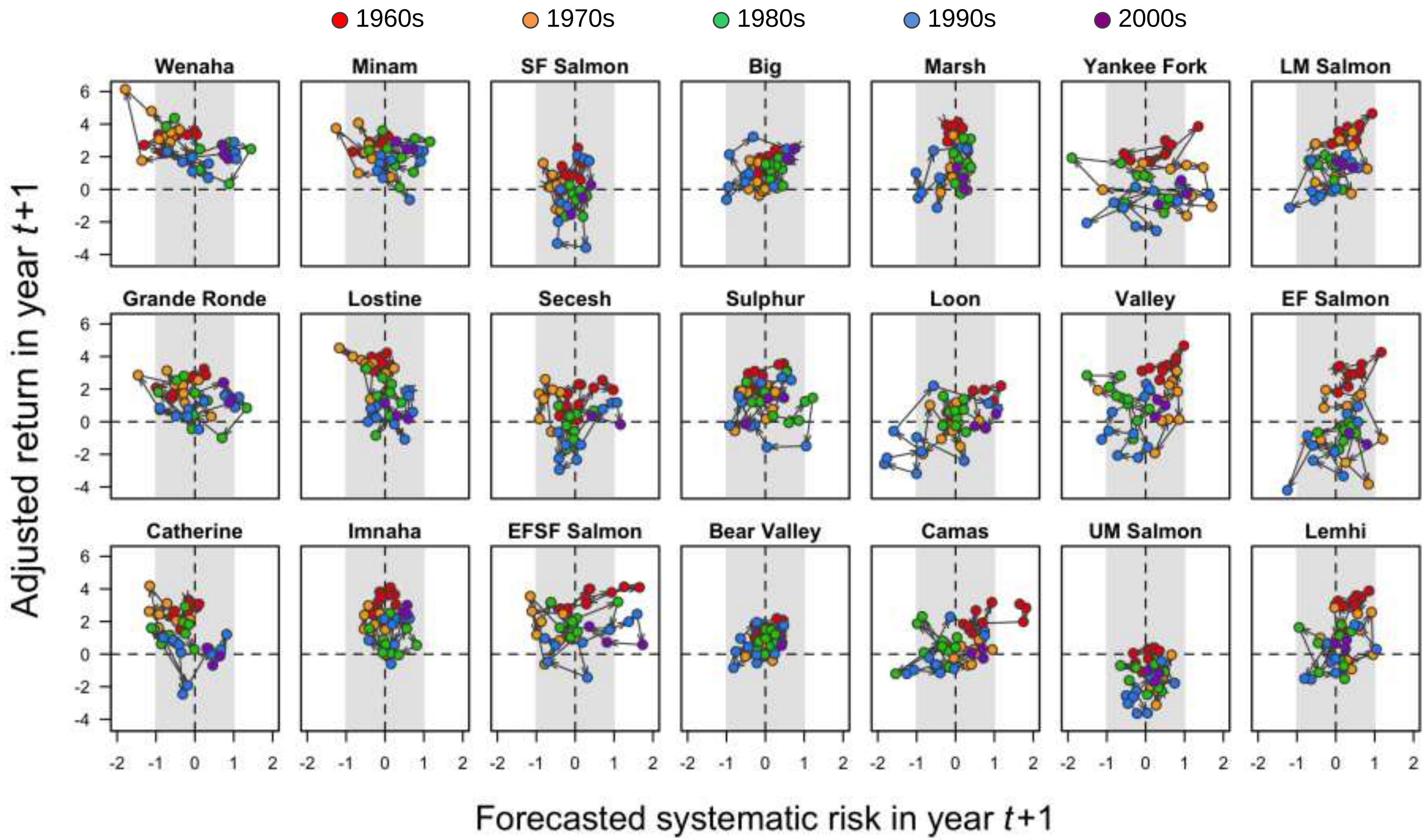
Market = PDO

Results – time series of betas



Market = PDO

Results – CAPM

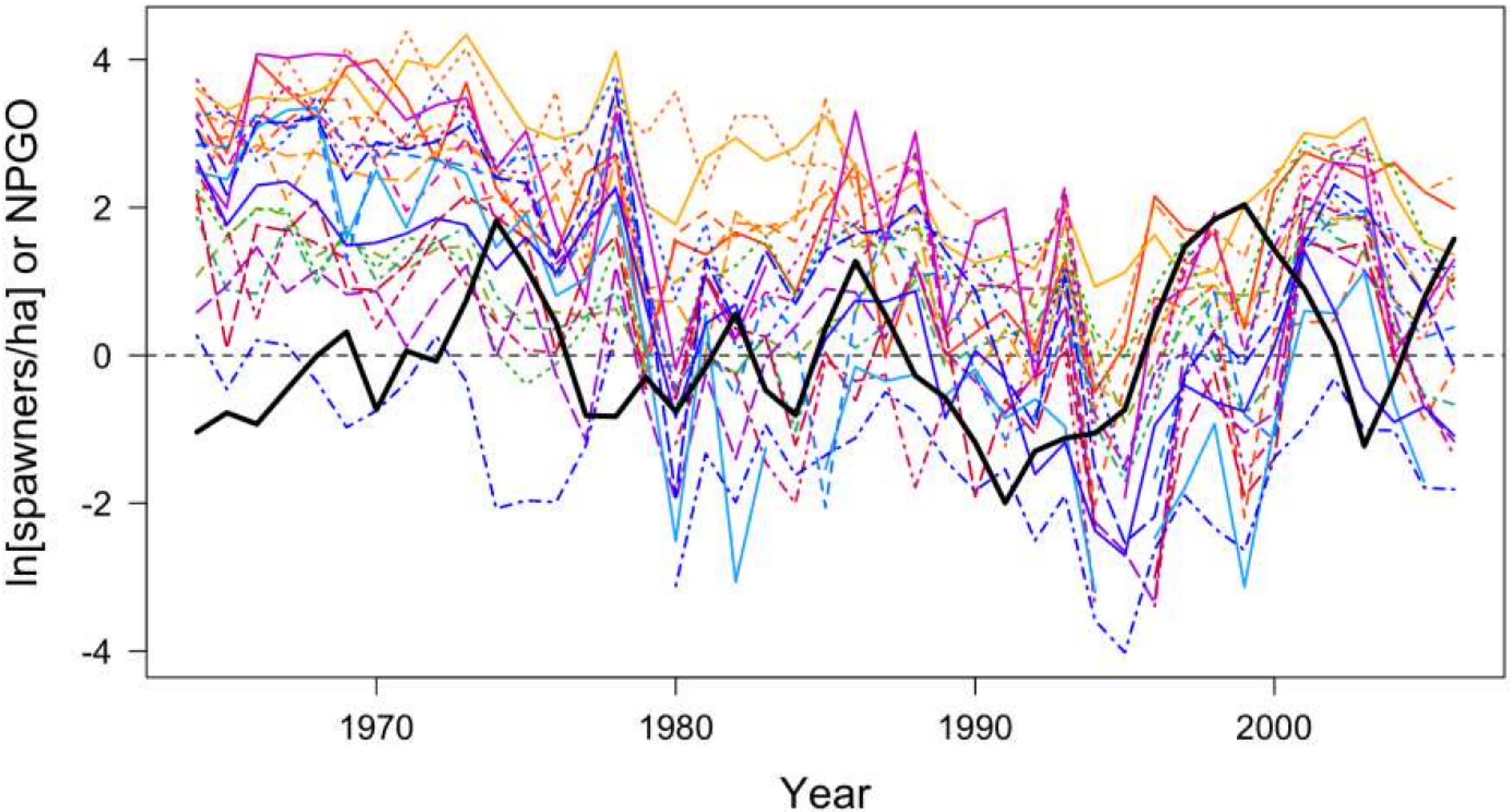


Market = PDO; *Risk-free* = 0

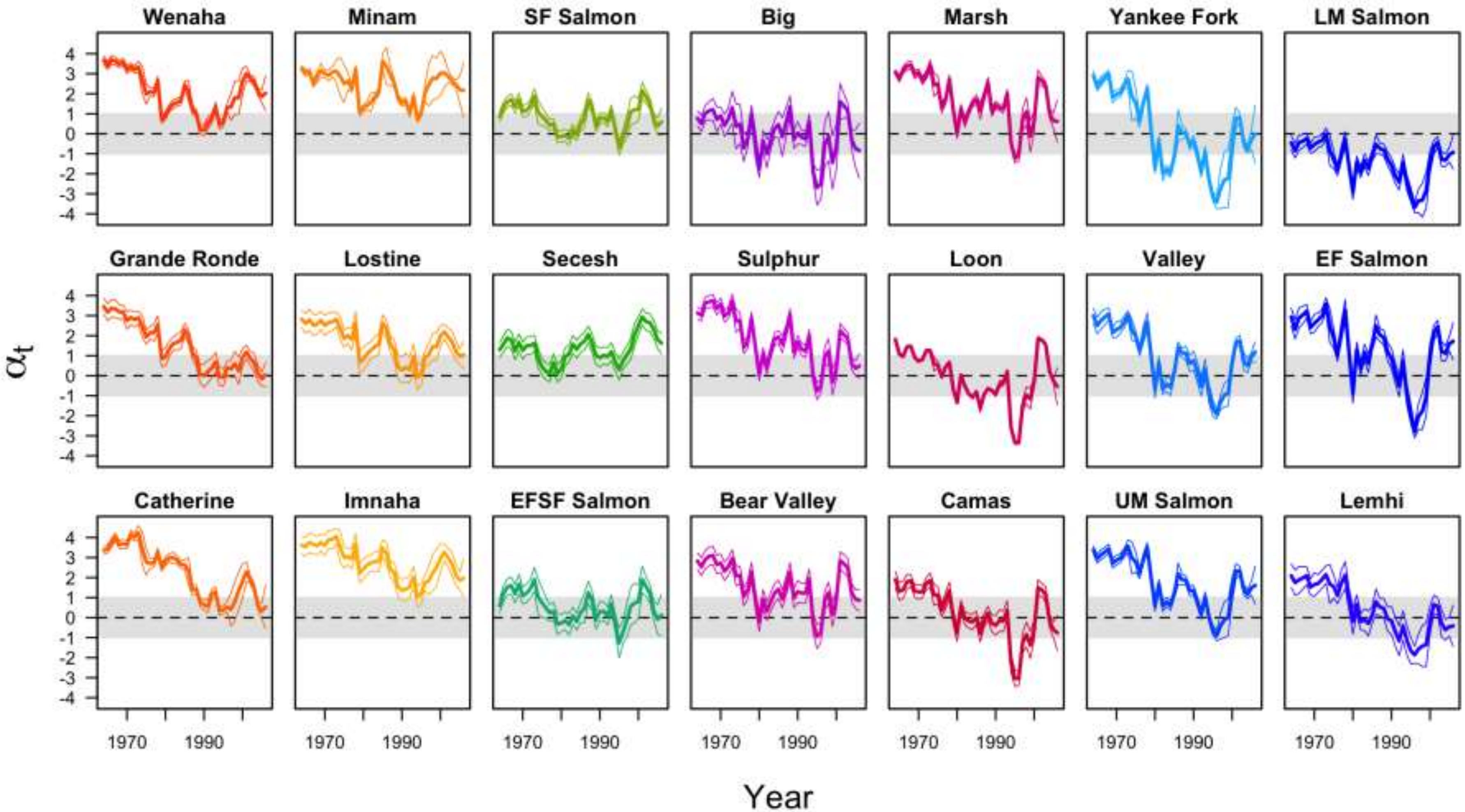
Next steps…

- Choose other market & risk-free indices
- Expand analyses to other listed ESUs
- Expand analyses to non-listed, exploited stocks
- Move toward portfolio selection

Time series of returns & NPGO

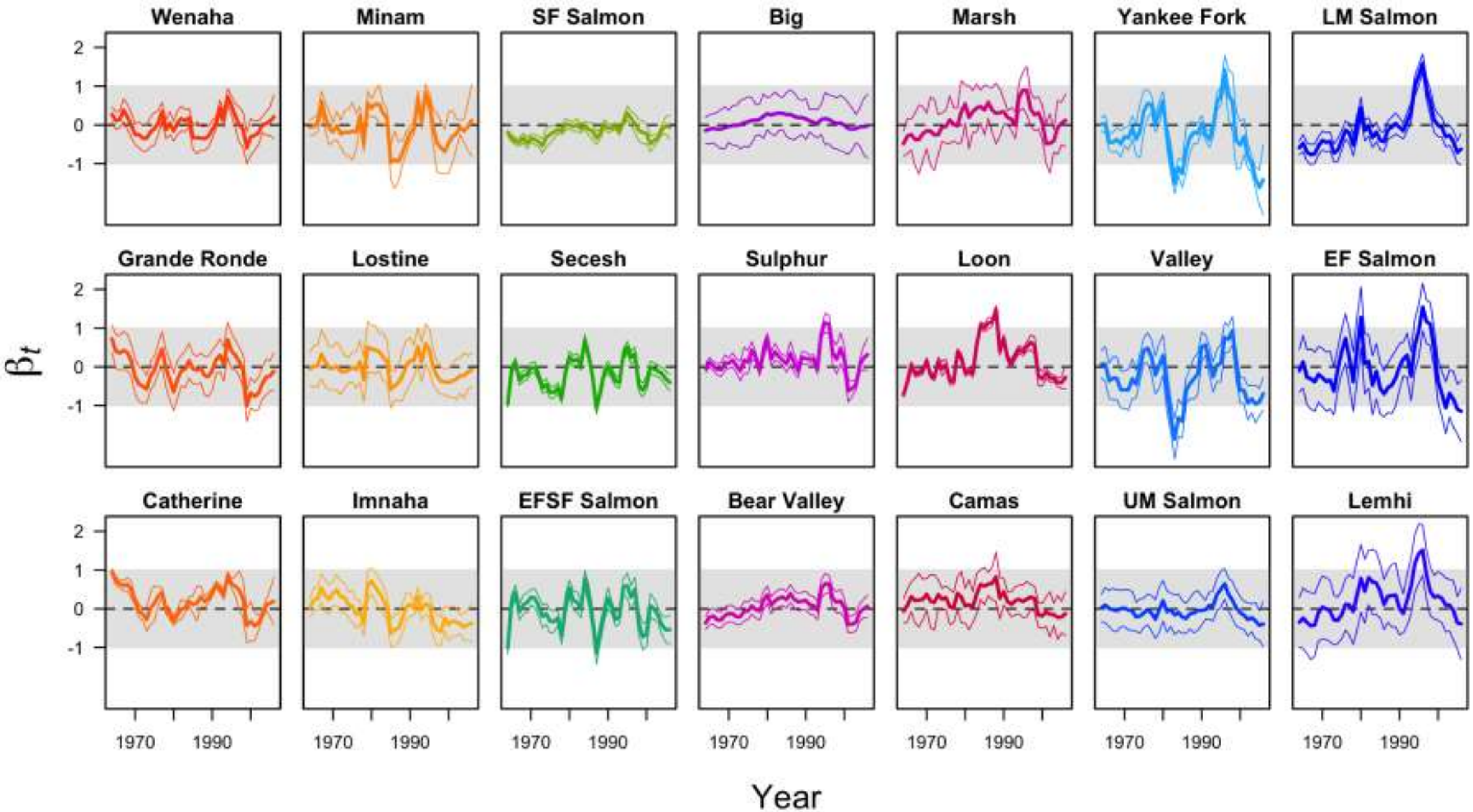


Results – ts of alphas (A)



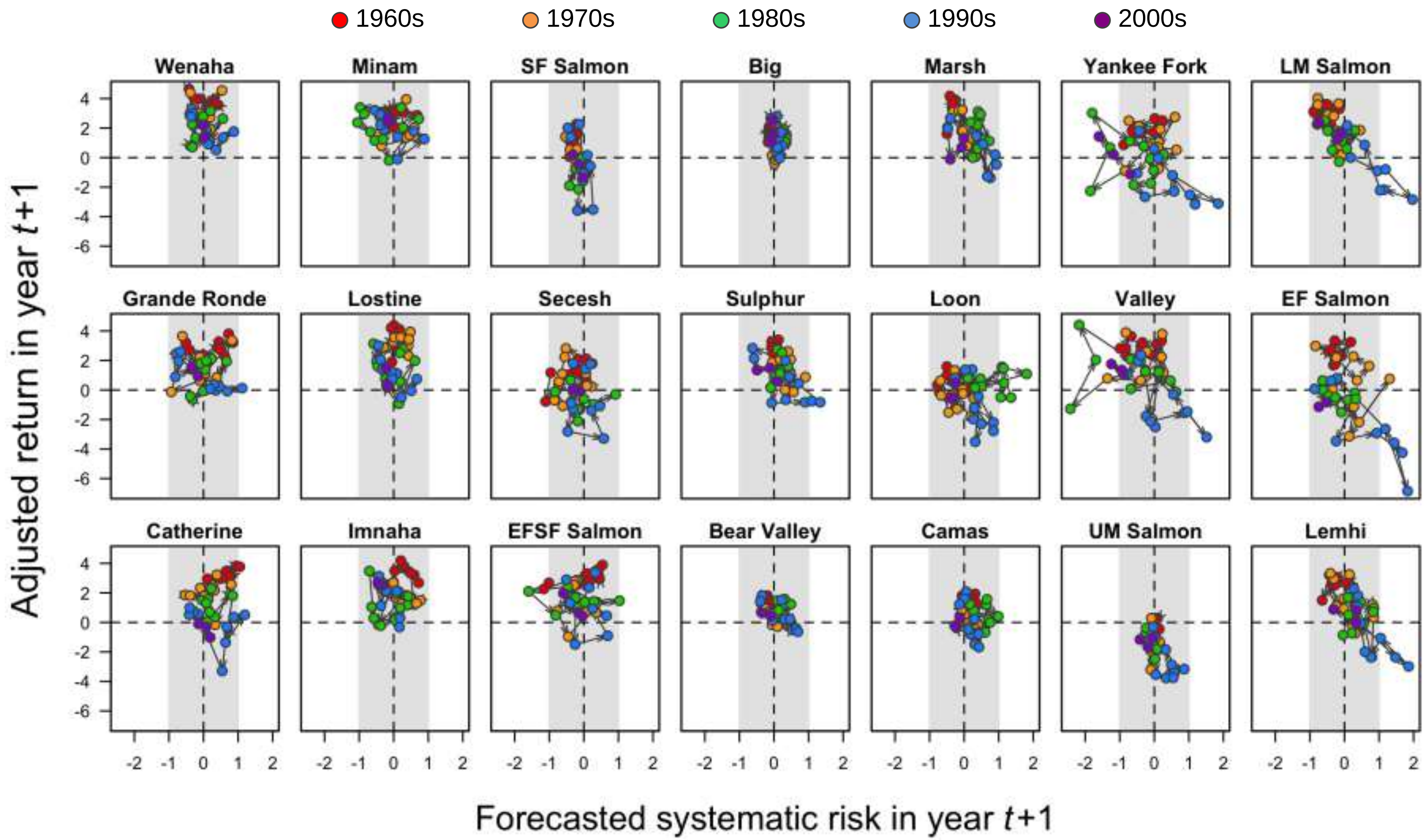
Market = NPGO

Results – ts of betas (A)



Market = NPGO

Results – CAPM ($r_f = 0$) (A)



Market = NPGO; *Risk-free* = 0